1. State the $\varepsilon-N$ definition of what $a_n \to 3$ means.

2. State the definition of what it means to say $\sum_{n=1}^{\infty} a_n = 10$. You may assume the definition of the limit of a sequence is already known.

3. State whether the following series converge absolutely, converge conditionally, or diverge. Justify briefly.
   
   (a) $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$
   (b) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$
   (c) $\sum_{n=3}^{\infty} \frac{n}{n^3 + 5}$
   (d) $\sum_{n=1}^{\infty} ne^{-n}$

4. What conclusion do you get from the ratio test on the convergence of $\sum_{n=1}^{\infty} \frac{1}{n^2}$?

5. What is the interval of convergence for the following power series? Justify your answers.
   
   (a) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$
   (b) $\sum_{n=1}^{\infty} \frac{(3x + 1)^n}{n}$

6. Use the fact that the geometric series $\sum_{n=0}^{\infty} x^n$ converges for $|x| < 1$ to find a power series to represent the following functions. State the radius of convergence for each.
   
   (a) $f(x) = \frac{1}{4-x}$
   (b) $g(x) = \int \frac{1}{1-x^4} dx$. Assume $g(0) = 3$.

7. Find the Taylor polynomial $T_3(x)$ for $f(x) = \frac{1}{x}$ expanded around $a = 1$. Show your work.

8. If $\sum_{n=1}^{\infty} a_n = 10$ and $s_n = a_1 + \cdots + a_n$, then
   
   (a) what is $\lim_{n \to \infty} s_n$
   (b) what is $\lim_{n \to \infty} a_n$?
9. True or False? If false, justify with a counterexample. If true, give a brief justification of why it is true.

(a) If \( \sum_{n=1}^{\infty} c_n(x - 2)^n \) converges for \( x = 0 \), then the series also converges for \( x = 3 \).

(b) If \( 0 \leq a_n \leq b_n \) and \( \sum_{n=1}^{\infty} b_n \) diverges implies \( \sum_{n=1}^{\infty} a_n \) diverges.

10. Find the first 5 coefficients \( (c_0, c_1, c_2, c_3, c_4) \) in the power series for \( (\sum_{n=0}^{\infty} \frac{x^n}{n!})(\sum_{n=0}^{\infty} \frac{x^n}{n!}) \).

11. How big must \( N \) be in order to make sure that the finite sum \( \sum_{n=1}^{N} \frac{(-1)^{n+1}}{n} \) approximates the infinite sum \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \) to within 0.01? Explain.

12. The Taylor series for the function \( f(x) = e^x \) is known to be \( \sum_{n=0}^{\infty} \frac{x^n}{n!} \). Use the Taylor remainder formula to determine the left hand endpoint \( a \) of an interval \( (a, 0) \) so that the finite sum \( 1 + x + \frac{x^2}{2!} \) approximates \( e^x \) to within \( \frac{1}{10} \times 10^{-3} \) on the whole interval \( (a, 0) \).

13. If \( 0 \leq b_n \leq a_n \) for \( n = 1, 2, 3, \ldots \), and \( \sum_{n=1}^{\infty} a_n = 100 \), prove that \( \sum_{n=1}^{\infty} b_n \) converges.

14. Sketch and label any 2 level curves for \( f(x, y) = x^2 + y^2 \).