Math 1297: Calculus II. Test 3 Practice Problems. Prof. Bruce Peckham

1. State the ϵ-N definition of what $a_n \to 3$ means.

2. State the definition of what it means to say $\sum_{n=1}^{\infty} a_n = 10$. You may assume the definition of the limit of a sequence is already known.

3. State whether the following series converge absolutely, converge conditionally, or diverge. Justify briefly.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$

   (c) $\displaystyle\sum_{n=3}^{\infty} \frac{n}{n^3+5}$

   (d) $\displaystyle\sum_{n=1}^{\infty} ne^{-n}$

4. What conclusion do you get from the ratio test on the convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$?

5. What is the interval of convergence for the following power series? Justify your answers.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(3x+1)^n}{n}$

6. Use the fact that the geometric series $\displaystyle\sum_{n=0}^{\infty} x^n$ converges for $|x| < 1$ to find a power series to represent the following functions. State the radius of convergence for each.

   (a) $f(x) = \dfrac{1}{4-x}$

   (b) $g(x) = \displaystyle\int \frac{1}{1-x^4}dx$. Assume $g(0) = 3$.

7. Find the Taylor polynomial $T_3(x)$ for $f(x) = \frac{1}{x}$ expanded around $a = 1$. Show your work.

8. If $\displaystyle\sum_{n=1}^{\infty} a_n = 10$ and $s_n = a_1 + \cdots + a_n$, then

   (a) what is $\displaystyle\lim_{n\to\infty} s_n$

   (b) what is $\displaystyle\lim_{n\to\infty} a_n$?

9. True or False? If false, justify with a counterexample. If true, give a brief justification of why it is true.

   (a) If $\sum_{n=1}^{\infty} c_n(x-2)^n$ converges for $x = 0$, then the series also converges for $x = 3$.

   (b) If $0 \le a_n \le b_n$ and $\sum_{n=1}^{\infty} b_n$ diverges implies $\sum_{n=1}^{\infty} a_n$ diverges.

10. Find the first 5 coefficients $(c_0, c_1, c_2, c_3, c_4)$ in the power series for $(\sum_{n=0}^{\infty} \frac{x^n}{n!})(\sum_{n=0}^{\infty} \frac{x^n}{n!})$.

11. How big must N be in order to make sure that the finite sum $\sum_{n=1}^{N} \frac{(-1)^{n+1}}{n}$ approximates the infinite sum $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ to within 0.01? Explain.

12. The Taylor series for the function $f(x) = e^x$ is known to be $\sum_{n=0}^{\infty} \frac{x^n}{n!}$. Use the Taylor remainder formula to determine the left hand endpoint a of an interval $(a, 0)$ so that the finite sum $1 + x + \frac{x^2}{2!}$ approximates e^x to within $\frac{1}{6} \times 10^{-3}$ on the whole interval $(a, 0)$.

13. If $0 \le b_n \le a_n$ for $n = 1, 2, 3, ...$, and $\sum_{n=1}^{\infty} a_n = 100$, prove that $\sum_{n=1}^{\infty} b_n$ converges.

14. Sketch and label any 2 level curves for $f(x, y) = x^2 + y^2$.