Practice problems for the Final Exam from Sections 15.2.

1. Let \( f(x, y) = \frac{2x^2 + 3y^2}{x^2 + y^2} \). Compute \( \lim_{(x,y) \to (0,0)} f(x, y) \) along the following paths:

   (a) the positive x-axis
   (b) the negative y-axis
   (c) along any line \( y = mx \)

   What can you conclude about \( \lim_{(x,y) \to (0,0)} f(x, y) \)?

2. Let \( f(x, y) = \frac{x^2 y}{x^4 + y^2} \). Compute \( \lim_{(x,y) \to (0,0)} f(x, y) \) along the following paths:

   (a) along any line \( y = mx \)
   (b) along the parabolas \( y = cx^2 \)

   What can you conclude about \( \lim_{(x,y) \to (0,0)} f(x, y) \)?

Answers:

1. (a) 2 (In fact, \( f(x,0) = 2 \) for any \( x \).)

   (b) 3

   (c) \( \frac{2+3m^2}{1+m^2} \)

   The limit does not exist. (As soon as limits along any two paths are not equal, the limit cannot exist. So parts a and b are enough to conclude the limit does not exist.)

2. (a) zero, no matter what \( m \) is (Replace \( y \) with \( mx \), divide numerator and denominator by \( x^2 \), and take the limit as \( x \to 0 \).

   (b) \( \frac{c}{1+c^2} \) (Replace \( y \) with \( cx^2 \), cancel \( x^4 \). No \( x \)'s remain, so \( f \) is “constant along parabolas.”)

   The limit does not exist (even though the limit along any straight line is zero!!)