Math 1597, Honors Calculus II

Proofs (and hints) to know for the Final Exam

1. Show the inverse derivative formula (Theorem 2.3, Chapter 7): ${f^{-1}}'(x) = \frac{1}{f'(f^{-1}(x))}$. (Hint: start with $f(f^{-1}(x)) = x$ and take the derivative of both sides, using the chain rule on the left side: $f'(f^{-1}(x)){f^{-1}}'(x) = 1$. Solve for the term ${f^{-1}}'(x)$. This is the same as the "alternative proof" in the text on p. 445.)

2. Show $\frac{d}{dx}e^x = e^x$, assuming you know that $\frac{d}{dx}\ln(x) = \frac{1}{x}$. (Hint: Special case of 1. Start with $\ln(e^x) = x$, differentiate, simplify.)

3. Geometric series: Prove that if $|x| < 1$, then $\sum_{n=0}^{\infty} x^n = \dfrac{1}{1-x}$. See Ch. 9, p. 621.

4. Special cases of geometric series. Show $1 + 1/2 + 1/2^2 + ... = 2$, or $0.99999... = 1$.

5. Prove $\sum_{n=1}^{\infty} \dfrac{1}{n}$ diverges. Use the ideas in the integral test, but do not directly quote the integral test. You must show that the sequence of partial sums $\{s_n\}$ diverges to infinity. See Sec. 9.3, p. 629-30.

6. Prove $\sum_{n=1}^{\infty} \dfrac{1}{n^2}$ diverges. Use the ideas in the integral test, but do not directly quote the integral test. You must show that the sequence of partial sums $\{s_n\}$ converges by showing the sequence is i) increasing, ii) bounded above. Then by the fact that any increasing sequence that is bounded above must converge (to its least upper bound). See Sec. 9.3, p. 629.

7. k^{th} term test. Show that if $\sum a_k$ converges, then $a_k \to 0$. (See proof in text on p. 623-4.)

8. Prove the basic comparison test: If $0 \le a_k \le b_k$, and $\sum_{n=1}^{\infty} b_k$ converges, then $\sum_{n=1}^{\infty} a_k$ converges. You must show exactly the same things as in the proof that $\sum_{n=1}^{\infty} \dfrac{1}{n^2}$ converges above. The reasons for the partial sums being bounded are, of course, different. See the text, p. 633, Theorem 3.3, part i.