1. State whether the following series converge absolutely, converge conditionally, or diverge. Give a brief justification of each.

(a) (5pts) \[ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \]

(b) (5pts) \[ \sum_{n=1}^{\infty} \frac{n}{2n + 5} \]

(c) (5pts) \[ \sum_{n=3}^{\infty} \frac{n^2 + 1}{n^4 - 4} \]

(d) (5pts) \[ \sum_{n=1}^{\infty} \frac{2^n}{n^n} \]

(e) (5pts) \[ \sum_{n=1}^{\infty} \frac{e^n}{n!} \]

2. If \( \sum_{n=1}^{\infty} a_n = 10 \) and \( s_n = a_1 + \cdots + a_n \), then

(a) (2pts) what is \( \lim_{n \to \infty} a_n \)?

(b) (2pts) what is \( \lim_{n \to \infty} s_{n+1} \)?

3. (4 pts) Match surface above with one of the following functions. Circle the correct answer.

(a) \( f(x, y) = y(x + 1) \)

(b) \( f(x, y) = x(y + 1) \)

(c) \( f(x, y) = x^2 + y^2 \)

4. True or False? No justification required.
(a) (3pts) If \( \sum_{n=1}^{\infty} c_n (x+2)^n \) converges for \( x = 0 \), then \( \sum_{n=1}^{\infty} c_n (x+2)^n \) converges for \( x = -3 \).

(b) (3pts) \( \sum_{n=1}^{\infty} |a_n| \) converges implies \( \sum_{n=1}^{\infty} a_n \) converges.

(c) (3pts) The convergence or divergence of the p-series can be determined by the ratio test.

5. (4pts) If \( f(x) = 1 + \frac{x}{2} + \frac{x^2}{3} + \frac{x^3}{4} + \ldots \) then what are the first 4 nonzero terms in a power series for \( f'(x) \)?

6. (5pts) By drawing a picture, rank the following from smallest to largest

(a) \( \int_{1}^{5} \frac{1}{x} \, dx \)

(b) \( \int_{2}^{6} \frac{1}{x} \, dx \)

(c) \( \sum_{n=2}^{5} \frac{1}{n} \)

7. Find the interval of convergence for the following power series:

(a) (7pts) \( \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n+1} \)

(b) (7pts) \( \sum_{n=1}^{\infty} \frac{(2x+1)^n}{n!} \)

8. (7pts) Use the fact that the geometric series \( \sum_{n=0}^{\infty} x^n \) converges to \( \frac{1}{1-x} \) for \( |x| < 1 \) to find a power series to represent the following functions. State the radius of convergence.

\[ f(x) = \frac{x^2}{10 - 3x} \]

9. (7pts) Find the Taylor polynomial \( T_3(x) \) for \( f(x) = \sqrt{x} \) expanded around \( a = 9 \). Show your work.

10. (7pts) How big must \( N \) be in order to make sure that the finite sum \( s_N = \sum_{n=1}^{N} \frac{(-1)^{n+1} x^n}{n^2} \) approximates the infinite sum \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n^2} \) to within 0.01? Explain.

11. (7pts) The Taylor series for the function \( f(x) = \ln(x+1) \) is known to be \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n} \).

Assume it is known that \( |f^{(3)}(x)| \leq \frac{27}{4} \) for \( -\frac{1}{3} \leq x \leq \frac{1}{3} \). (It is true.) Use the Taylor remainder formula to determine an upper bound on the error in using the finite sum \( x - \frac{x^2}{2} \) to approximate \( \ln(x+1) \) for \( -\frac{1}{3} \leq x \leq \frac{1}{3} \). You need not simplify your answer.

12. (5pts) Extra Credit. Explain where the \( \frac{27}{4} \) in the previous problem came from.