Math 1597, Honors Calculus II: Test 3 practice problems, excluding proofs. Bruce Peckham

1. State whether the following series converge absolutely, converge conditionally, or diverge. Give a brief justification of each.

   (a) (5pts) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

   (b) (5pts) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2n+5}$

   (c) (5pts) $\displaystyle\sum_{n=3}^{\infty} \frac{n^2+1}{n^4-4}$

   (d) (5pts) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^n}$

   (e) (5pts) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n!}$

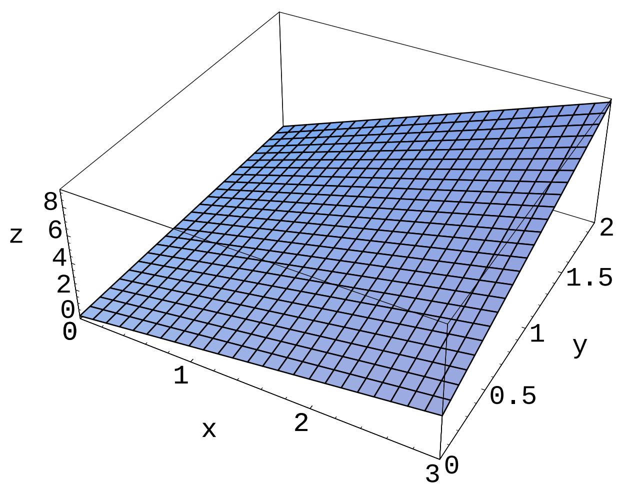
2. If $\displaystyle\sum_{n=1}^{\infty} a_n = 10$ and $s_n = a_1 + \cdots + a_n$, then

   (a) (2pts) what is $\displaystyle\lim_{n\to\infty} a_n$?

   (b) (2pts) what is $\displaystyle\lim_{n\to\infty} s_{n+1}$?

3. (4 pts) Match surface above with one of the following functions. Circle the correct answer.

   (a) $f(x,y) = y(x+1)$
   (b) $f(x,y) = x(y+1)$
   (c) $f(x,y) = x^2 + y^2$



4. True or False? No justification required.

(a) (3pts) If $\sum_{n=1}^{\infty} c_n(x+2)^n$ converges for $x=0$, then $\sum_{n=1}^{\infty} c_n(x+2)^n$ converges for $x=-3$.

(b) (3pts) $\sum_{n=1}^{\infty} |a_n|$ converges implies $\sum_{n=1}^{\infty} a_n$ converges.

(c) (3pts) The convergence or divergence of the p-series can be determined by the ratio test.

5. (4pts) If $f(x) = 1 + \frac{x}{2} + \frac{x^2}{3} + \frac{x^3}{4} + ...$ then what are the first 4 nonzero terms in a power series for $f'(x)$?

6. (5pts) By drawing a picture, rank the following from smallest to largest

(a) $\int_1^5 \frac{1}{x} dx$

(b) $\int_2^6 \frac{1}{x} dx$

(c) $\sum_{n=2}^{5} \frac{1}{n}$

7. Find the interval of convergence for the following power series:

(a) (7pts) $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n+1}$

(b) (7pts) $\sum_{n=1}^{\infty} \frac{(2x+1)^n}{n!}$

8. (7pts) Use the fact that the geometric series $\sum_{n=0}^{\infty} x^n$ converges to $\frac{1}{1-x}$ for $|x| < 1$ to find a power series to represent the following functions. State the radius of convergence. $f(x) = \frac{x^2}{10-3x}$

9. (7pts) Find the Taylor polynomial $T_3(x)$ for $f(x) = \sqrt{x}$ expanded around $a = 9$. Show your work.

10. (7pts) How big must N be in order to make sure that the finite sum $s_N = \sum_{n=1}^{N} \frac{(-1)^{n+1}}{n^2}$ approximates the infinite sum $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$ to within 0.01? Explain.

11. (7pts) The Taylor series for the function $f(x) = \ln(x+1)$ is known to be $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{n}$. Assume it is known that $|f^{(3)}(x)| \le \frac{27}{4}$ for $-\frac{1}{3} \le x \le \frac{1}{3}$. (It is true.) Use the Taylor remainder formula to determine an upper bound on the error in using the finite sum $x - \frac{x^2}{2}$ to approximate $\ln(x+1)$ for $-\frac{1}{3} \le x \le \frac{1}{3}$. You need not simplify your answer.

12. (5pts) Extra Credit. Explain where the '$\frac{27}{4}$' in the previous problem came from.