Math 1597, Honors Calculus II

Test 3 practice problems answers as of April 30, 2008

1. (a) converges absolutely (since $\sum \frac{1}{n^2}$ converges)
   (b) diverges by nth term test since $a_n \to \frac{1}{2} \neq 0$
   (c) conveges absolutely by limit comparison test with $\frac{1}{n^2}$
   (d) converges absolutely by root test ($|a_n|^{\frac{1}{n}} = \frac{2}{n} \to 0 < 1$). Ratio test also works, but with more effort.
   (e) Converges absolutely by ratio test: $\frac{|a_{n+1}|}{|a_n|} = \frac{e}{n+1} \to 0 < 1$.
2. (a) 0 (the individual terms must go to zero for any series that converges)
   (b) 10 (the series converging to 10 means by definition that $s_n \to 10$. s_{n+1} has the same limit as s_n, so s_{n+1} must also converge to 10.
3. (b). (Check the trace at $y = 0$ - or lots of other ways.)
4. (a) True. ($x = -3$ is closer to -2 than is $x = 0$.)
   (b) True. Absolute convergence does imply convergence.
   (c) False. The ratio test always fails for p-series.
5. $f'(x) = \frac{1}{2} + \frac{2x}{3} + \frac{3x^2}{4} + \frac{4x^3}{5} + ...$
6. $(b) \leq (c) \leq (a)$ (Picture should be drawn representing each as an area.)
7. (a) $(-1, 1]$ (Use ratio test and check endpoints.)
   (b) $(-\infty, \infty)$ (Use ratio test.)
8. $\dfrac{x^2}{10} \displaystyle\sum_{n=1}^{\infty} (\frac{3x}{10})^n$
9. $T_3(x) = 3 + \frac{1}{6}(x-9) - \frac{1}{216}(x-9)^2 + \frac{1}{3888}(x-9)^3$
10. $N \geq 9$ (Need justification, including that you are using the Alternating Series Test for error: $|s - s_n| < |a_{n+1}|$.)
11. $\frac{1}{24}$
12. Extra Credit $M = \max |f'''(x)|$ for $x \in (-\frac{1}{3}, \frac{1}{3})$. Since $f'''(x) = \frac{2}{(x+1)^3}$ is decreasing, its max is at its lefthand endpoint, $x = -\frac{1}{3}$, and $f'''(-\frac{1}{3}) = \frac{27}{4}$.