Math 1597, Honors Calculus II
Test 3 practice problems answers as of April 30, 2008

1. (a) converges absolutely (since \( \sum \frac{1}{n^2} \) converges)
   (b) diverges by nth term test since \( a_n \to \frac{1}{2} \neq 0 \)
   (c) converges absolutely by limit comparison test with \( \frac{1}{n^2} \)
   (d) converges absolutely by root test (\( |a_n|^{\frac{1}{n}} = \frac{2}{n} \to 0 < 1 \)). Ratio test also works, but with more effort.
   (e) Converges absolutely by ratio test: \( \frac{|a_{n+1}|}{|a_n|} = \frac{e}{n+1} \to 0 < 1. \)

2. (a) 0 (the individual terms must go to zero for any series that converges)
   (b) 10 (the series converging to 10 means by definition that \( s_n \to 10 \). \( s_{n+1} \) has the same limit as \( s_n \), so \( s_{n+1} \) must also converge to 10.

3. (b). (Check the trace at \( y = 0 \) - or lots of other ways.)

4. (a) True. (\( x = -3 \) is closer to \( -2 \) than is \( x = 0. \))
   (b) True. Absolute convergence does imply convergence.
   (c) False. The ratio test always fails for \( p \)-series.

5. \( f'(x) = \frac{1}{2} + \frac{2x}{3} + \frac{3x^2}{4} + \frac{4x^3}{5} + ... \)

6. (b) \( \leq (c) \leq (a) \) (Picture should be drawn representing each as an area.)

7. (a) \( (-1, 1] \) (Use ratio test and check endpoints.)
   (b) \( (-\infty, \infty) \) (Use ratio test.)

8. \( \frac{x^2}{10} \sum_{n=1}^{\infty} \frac{3x}{10^n} \)

9. \( T_3(x) = 3 + \frac{1}{6}(x - 9) - \frac{1}{216}(x - 9)^2 + \frac{1}{3888}(x - 9)^3 \)

10. \( N \geq 9 \) (Need justification, including that you are using the Alternating Series Test for error: \( |s - s_n| < |a_{n+1}|. \))

11. \( \frac{1}{27} \)

12. Extra Credit \( M = \max |f'''(x)| \) for \( x \in (-\frac{1}{3}, \frac{1}{3}) \). Since \( f'''(x) = \frac{2}{(x+1)^3} \) is decreasing, its max is at its lefthand endpoint, \( x = -\frac{1}{3} \), and \( f'''(\frac{-1}{3}) = \frac{27}{4} \).