Name______________________________________
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Math 1297: Calculus II
Test 1
Prof. Bruce Peckham
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Unless otherwise noted, each part of each problem is worth 6 points.

1. (5pts) Solve exactly (without calculator) for x: $e^{10x} = \log_{10}(2)$. Do not simplify.

2. Find the derivatives of the following functions. Show all work. Do not use a calculator. You need not simplify your answers.

   (a) (4pts) e^{x^2+1}

   (b) (5pts) $f(\theta) = \sin^{-1}(10^{\theta})$.

   (c) (6 pts) $x^{\sin(x)}$ Assume $x > 0$. (Hint: Use logarithmic differentiation.) Write your final answer in terms of x.

3. (6pts) Sketch a picture of any two vectors $\vec{a}$ and $\vec{b}$ in the plane having the property that $\vec{a} \bullet \vec{b} < 0$. On the same picture, sketch and label $proj_{\vec{b}}\vec{a}$.

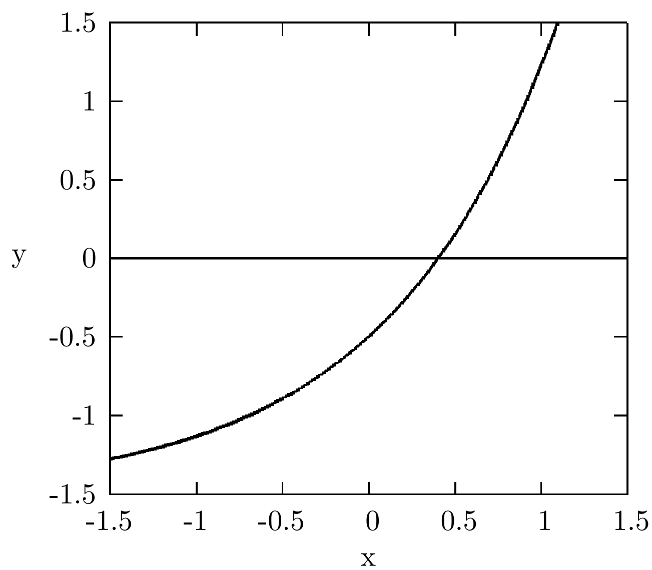
4. (4pts) Let $P = (0, -3, 2)$ and $Q = (5, 1, -1)$. What is the distance between the two points? Give an exact answer, not a calculator approximation.

5. (5pts) Convert the equation of the following line from its vector parametric form to the symmetric form:

$$\vec{r}(t) = < 3, -2, 1 > + t < 4, 5, 6 >$$

6. (4pts) Circle ALL of the following equations that define surfaces in space which DO NOT intersect the yz plane at all.

   A. $x^2 + y^2 + z^2 = 1$ B. $x^2 - y^2 + z^2 = 1$ C. $x^2 + y^2 - z^2 = 1$ D. $x^2 - y^2 - z^2 = 1$



7. (5pts) The graph of a function is given in the accompanying figure. Sketch the graph of its inverse on the same figure.

8. (5pts) Assume the following: $f(1) = 2, f^{-1\prime}(2) = 3$. What does this imply about the derivative of the original function f? Suggestion: Draw a figure.

   A. $f'(2) = \frac{1}{3}$ B. $f'(2) = -\frac{1}{3}$ C. $f'(1) = \frac{1}{3}$ D. $f'(1) = -\frac{1}{3}$ E. $f'(3) = 2$

9. (10pts) What is the equation of the plane which contains the three points: $(3, -1, 2), (2, 0, 0)$, and $(0, 5, 3)$

10. (3 pts each; 24 pts total) True-False. Write TRUE or FALSE answers to the left of each problem. No justification necessary. No partial credit.

    (a) If x and y are both positive real numbers, $\ln(x \cdot y) = \ln(x) + \ln(y)$

    (b) $\lim_{x\to\infty}(e^{-x^2}) = 0$.

    (c) $\frac{d}{dx}(\pi^x) = x\pi^{x-1}$

    (d) The cross product of two unit vectors is always another unit vector.

    (e) Using cylindrical coordinates in space, the equation $r = 3$ represents a circle.

    (f) The following expression is meaningful: $\vec{a} \bullet (\vec{b} \bullet \vec{c})$.

    (g) The vector between any two points in a plane is always perpendicular to the normal to the plane.

    (h) $\vec{a} \bullet \vec{a} = |\vec{a}|^2$

11. (5pts) Evaluate $\int \frac{r}{r^2 - 5} dr$

12. (7 pts) Show that $\frac{d}{dx}(\arctan(x)) = \frac{1}{x^2+1}$ using the fact that $\arctan(x)$ is the inverse of the $\tan(x)$ (restricted to $(-\pi/2, \pi)$) and the derivative of $\tan(x)$ is $\sec^2(x)$.

13. (5pts) Show that $\frac{d}{dx}(2^x) = 2^x \ln(2)$ using the fact that the derivative of e^x is e^x.

14. (Extra credit 5pts) Assume that two planes intersect in a line. If you know the equations of the two planes (in the standard form: $Ax + By + Cz = D$), explain how you would obtain a direction vector for the line of intersection. Do not do any computations!

Formulas:

Inverse trig function derivatives:

$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\cos^{-1}(x)) = -\frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$
$\frac{d}{dx}(\csc^{-1}(x)) = -\frac{1}{x\sqrt{x^2-1}}$	$\frac{d}{dx}(\sec^{-1}(x)) = \frac{1}{x\sqrt{x^2-1}}$	$\frac{d}{dx}(\cot^{-1}(x)) = -\frac{1}{1+x^2}$

Hyperbolic function derivatives:

$\frac{d}{dx}(\sinh(x)) = \cosh(x)$	$\frac{d}{dx}(\cosh(x)) = \sinh(x)$	$\frac{d}{dx}(\tanh(x)) = \text{sech}^2(x)$
$\frac{d}{dx}(\text{csch}(x)) = -\text{csch}(x)\coth(x)$	$\frac{d}{dx}(\text{sech}(x)) = -\text{sech}(x)\tanh(x)$	$\frac{d}{dx}(\coth(x)) = -\text{csch}^2(x)$

Inverse hyperbolic function derivatives:

$\frac{d}{dx}(\sinh^{-1}(x)) = \frac{1}{\sqrt{1+x^2}}$	$\frac{d}{dx}(\cosh^{-1}(x)) = \frac{1}{\sqrt{x^2-1}}$	$\frac{d}{dx}(\tanh^{-1}(x)) = \frac{1}{1-x^2}$
$\frac{d}{dx}(\text{csch}^{-1}(x)) = -\frac{1}{\lvert x\rvert\sqrt{x^2+1}}$	$\frac{d}{dx}(\text{sech}^{-1}(x)) = -\frac{1}{x\sqrt{x^2-1}}$	$\frac{d}{dx}(\coth^{-1}(x)) = \frac{1}{1-x^2}$

Projections:

$\text{comp}_{\vec{a}}\vec{b} = \vec{b} \cdot \frac{\vec{a}}{\lvert\vec{a}\rvert}$	$\text{proj}_{\vec{a}}\vec{b} = (\vec{b} \cdot \frac{\vec{a}}{\lvert\vec{a}\rvert})\frac{\vec{a}}{\lvert\vec{a}\rvert}$