Math 1297, Calculus II

Lecture Section 8

Proofs (and hints) to know for Test 1

1. Show $\vec{a} \times \vec{b}$ is perpendicular to $\vec{a}$. (Hint: Dot $\vec{a} \times \vec{b}$ with $\vec{a}$ and show it equals zero. See p. 810.)

2. Show the inverse derivative formula (7.1): ${f^{-1}}'(x) = \frac{1}{f'(f^{-1}(x))}$. (Hint: start with $f(f^{-1}(x)) = x$ and take the derivative of both sides, using the chain rule on the left side: $f'(f^{-1}(x)){f^{-1}}'(x) = 1$. Solve for the term ${f^{-1}}'(x)$.)

3. Show $\frac{d}{dx}e^x = e^x$, assuming you know that $\frac{d}{dx}\ln(x) = \frac{1}{x}$. (Hint: Special case of 2. Start with $\ln(e^x) = x$, differentiate, simplify.)

4. Show $\frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}$. (Hint: Special case of 2. Start with $\tan(\arctan(x)) = x$, differentiate, simplify using the appropriate triangle..)

5. Show $\log_a(x) = \frac{\ln(x)}{\ln(a)}$. (Hint: $y = \log_a(x) \iff a^y = x$. Now take ln of both sides.)

6. Show $\frac{d}{dx}a^x = a^x \ln(a)$, assuming you know that $\frac{d}{dx}e^x = e^x$. (Hint: Rewrite a^x as $(e^{\ln(a)})^x = e^{x\ln(a)}$. Differentiate.)

7. Show $\ln(ab) = \ln(a) + \ln(b)$ directly from the definition of $\ln(x) = \int_1^x \frac{1}{t}dt$. (Hint: $\ln(ab) = \int_1^{ab} \frac{1}{t}dt = \int_1^a \frac{1}{t}dt + \int_a^{ab} \frac{1}{t}dt$. Show using a u-substitution that the last integral is equal to $\int_1^b \frac{1}{t}dt$.

8. Use the natural log property of $\ln(ab) = \ln(a) + \ln(b)$ to show that $e^{a+b} = e^a e^b$. (Hint: Show $\ln(e^{a+b}) = \ln(e^a e^b) : \ln(e^{a+b}) = (a+b)\ln(e) = a+b = a\ln(e) + b\ln(e) = \ln(e^a) + \ln(e^b) = \ln(e^a e^b)$.

9. Show $\frac{d}{dx}\cosh(x) = \sinh(x)$ (or vice versa). (Hint: Direct computation using the definition of $\sinh(x)$ and $\cosh(x)$.)