Math 1597 Test 2 practice problems. B. Peckham. March 31, 2008

1. Determine the following limits (if they exist):

   (a) $\lim_{x\to 0} \frac{e^x - x - 1}{x^2}$

   (b) $\lim_{x\to\infty} (\frac{x}{x+1})^x$

2. How many integrations by parts would be necessary to find $\int x^{10}\cos(3x)dx$? Explain briefly.

3. Do the following integral: $\int 3x\cos(2x)dx$.

4. Integrate $\int_0^{\pi/2} \sin^3(x)\cos(x)dx$ in two ways (using two different u substitutions).

5. Integrate $\int \cos(2x)\sin(5x)dx$ using the trig identitiy: $\sin(A)\cos(B) = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$.

6. Consider the following integral: $\int \frac{x^3}{\sqrt{4+x^2}}dx$. Make the substitution $x = 2\tan(\theta)$ to rewrite the integral with respect to θ. Simplify enough so that the integrand has no square roots in it, but DO NOT EVALUATE THE INTEGRAL.

7. Write the form of the partial fractional decomposition for each of the following. DO NOT SOLVE FOR THE CONSTANTS.

   (a) $\dfrac{1+x^2}{x(x^2+9)}$.

   (b) $\dfrac{2x^3 - 3x + 1}{x^3(x-2)}$.

8. Use the fact that $\dfrac{3x^2+4x-2}{x(x+1)(2x-1)} = \dfrac{2}{x} - \dfrac{1}{x+1} + \dfrac{1}{2x-1}$ to find $\displaystyle\int \frac{3x^2+4x-2}{x(x+1)(2x-1)}dx$.

9. Evaluate $\displaystyle\int \frac{6x+1}{3x+2}dx$

10. Consider the integral $\displaystyle\int \frac{\sqrt{x}}{x-4}dx$. Determine the integral with respect to u after making the substitution $u = \sqrt{x}$. Is the new integral any easier to integrate than the original one? Explain why briefly (but DON'T EVALUATE).

11. (5pts) The following integral comes from a table of integrals:

$$\int \frac{du}{u^2\sqrt{u^2-a^2}} = \frac{\sqrt{u^2-a^2}}{a^2u} + C.$$

    Use it to determine

$$\int \frac{dx}{x^2\sqrt{4x^2-3}}$$

12. Evaluate $\displaystyle\int_0^1 \frac{1}{\sqrt{x}}dx$ using the formal definition of the improper integral.

13. True or False? If $\sum_{n=1}^{\infty} a_n$ converges then $\{a_n\}_{n=1}^{\infty}$ must converge. Explain briefly.

14. Give an example of a sequence that is bounded but not monotonic.

15. What is $\lim_{n\to\infty} \dfrac{2n+500}{2n^2+1}$? Indicate your reasoning briefly.

16. Consider the following list of series.

    (a) $\sum_{n=2}^{\infty} \dfrac{2^{n+1}}{3^{n-1}}$

    (b) $-1 + \dfrac{1}{2} - \dfrac{1}{4} + \dfrac{1}{8} - \cdots$

    (c) $\sum_{n=1}^{\infty} \dfrac{n^2+5n}{2n^2+3}$

    (d) $\sum_{n=1}^{\infty} \dfrac{n^2+3n+1}{n^3-1}$

    - Which of the above series are geometric?
    - What is "r" for the series that are geometric?
    - Which of the above series converge? Justify briefly. No formal proof required.
    - To what number does each geometric series converge?

17. Circle the correct inequality. Justify with a sketch.

$$\int_0^5 e^{-x}dx \quad < > \leq \geq = \quad \sum_{n=0}^{4} e^{-n}.$$

18. Estimate an upper bound on the error in approximating $\sum_{n=1}^{\infty} \dfrac{1}{n^3}$ by $1 + \frac{1}{2^3} + ... + \frac{1}{10^3}$. Hint: use an integral.

19. Give the ϵ-N definition of what it means for a sequence a_n to converge to 10.

20. Give the definition of what it means for the series $\sum_{n=1}^{\infty} a_n$ to converge to 10.

21. Prove directly (by obtaining a formula for the partial sums s_n) that $\sum_{n=1}^{\infty} \dfrac{1}{3^{n-1}} = \dfrac{3}{2}$.

22. More proofs chosen from:

    (a) $\sum a_k$ converges implies $a_k \to 0$.

    (b) Basic comparison test easy case. Show partial sums are bounded!!! Don't forget to establish notation for partial sums.

    (c) Show that $1 - \frac{1}{n^2}$ converges to one. (ϵ-N proof)