Separation of variables Template

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Goal: Find an analytic solution to a separable DE: f(y)dy = g(t)dt

Example: y’=2y, y(0)=1. Solve this IVP and determine how long it takes for the initial amount to double.

Clear variables

\[
y = .
\]
\[
t = .
\]

Separate by hand first: 1/y dy = 2 dt

\[
f = \frac{1}{y}
\]
\[
\frac{1}{y}
\]
\[
g = 2
\]
\[
2
\]

Integrate both sides:

\[
\text{LHS} = \text{Integrate}[f, y]
\]\n\[
\log[y]
\]
\[
\text{RHS} = \text{Integrate}[g, t] + C
\]\n\[
C + 2 t
\]
\[
\text{gensln} = \text{Solve}[\text{RHS} = \text{LHS}, y]
\]\n\[
\{\{y \rightarrow e^{C+2 t}\}\}
\]

Trick to eliminate both sets of braces:

\[
\text{gensln} = y /. \text{gensln}[[1]][[1]]
\]\n\[
e^{C-2 t}
\]

Specify initial conditions:

\[
t0 = 0
\]\n\[
0
\]
\[
y0 = 3
\]\n\[
3
\]
Use the initial conditions to solve for the arbitrary constant.

\[
IC = Solve[y0 == gensln / . t \to t0, \ C]
\]

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found.

\[
\{\{C \to \text{Log}[3]\}\}
\]

Use the value of the constant in the general solution to obtain the particular solution.

\[
sln = gensln / . \text{IC}[[1]]
\]

\[
e^{2t}\text{Log}[3]
\]

Simplify[sln]

\[
3 \ e^{2t}
\]

\[
\{3 \ e^{2t}\}
\]

doubletime = Solve[sln == 2 + y0, t]

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found.

\[
\{\{t \to \frac{\text{Log}[2]}{2}\}\}
\]

\[
N[doubletime[[1]]]
\]

\[
\{t \to 0.346574\}
\]