Differential Equations and Linear Algebra
Math 3280
Lab #2: Introduction to Differential Equations
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Directions: Turn in a written lab report dealing with the tasks below. Your report should include goals, description of the procedures you used in the lab, *Mathematica* output with comments, and conclusions, as indicated from the "Lab Procedures and Guidelines" file. The writeup may be typed as *Mathematica* text, added neatly by hand to *Mathematica* output, or done with a word processor. Grading: Goals (G) 1, Procedures (Pro) 1, *Mathematica* tasks (Ma) 14 (details below), Conclusions (C) 2, Presentation and Organization (P+O) 2, Total 20.

Before starting the lab, reread the Mathematica Notes/Hints available from the link accompanying Lab 1. Do the following tasks with the help of *Mathematica.*

1. (8 pts) Slope field and qualitative graphical solution with varying amounts of help from *Mathematica*.
   (a) Sketch a slope field *by hand* for the differential equation $\frac{dx}{dt} = -0.5t$, and sketch *by hand* the solution to the initial value problem corresponding to the initial conditions $x(0) = 1$.
   (b) Create the slope field for $\frac{dx}{dt} = -0.5t$ with the following *Mathematica* command: VectorPlot[{1, -0.5 t}, {t,-2,2}, {x,-2,2}]. Why does the vector field {1,-0.5t} correspond to the slope field for the differential equation? Add the option VectorStyle → "Segment" to plot the slope marks without vector arrows.
   (c) Print out the vector field, and, *by hand*, sketch the solution to the differential equation corresponding to the initial condition $x(0) = 1$.
   (d) Use StreamPlot instead of VectorPlot to produce a graph of solutions which line up with the vector field. By hand, sketch or highlight the solution corresponding to the initial condition $x(0) = 1$.
   (e) Display the slope field and stream plot graphs simultaneously using the Show command. (Hint: name each plot first.)
   (f) Comment on the effort and accuracy of each of the above methods for obtaining a plot of the particular solution.
   (g) Experiment with parameters and the Manipulate command: graph a slope field for $\frac{dx}{dt} = at$, where a is "manipulated." Allow a to vary from at least -2 to 2. Add another manipulated variable to manipulate the size of the plot window.

2. (3pts) Calc II Template. A template for solving Calc II type initial value problems is provided with the Mathematica Notes provided with Lab 1. Modify this template to find a formula (analytic) solution to the initial value problem:

$$\frac{dx}{dt} = -0.5t, \quad x(0) = 1.$$

   Plot your solution. Compare it with the plot you obtained for the this same initial value problem using the slope field in part 1 of this lab.

3. (3pts) Verifying solutions to initial value problems. Consider the function:

$$f(x) = 3e^{-x/2}\cos(3x) + 2$$

   (a) Plot $f(x)$ over the interval $0 \leq x \leq 2\pi$. Example: Plot[x^2, {x,-1,1}]. More plotting features can be found by looking up the Plot command with the online Help.

(b) Repeat part (a) for $f'(x)$ and $f''(x)$.

(c) Obtain a simultaneous plot of f, f', and f'' over the interval $0 \leq x \leq 2\pi$. Example of Plot command: Plot[{xˆ2, .3 x, 1-x}, {x,-1.5,1}].

(d) Use Mathematica to show that $f(x)$ satisfies the initial value problem:

$$y''(x) + y'(x) + \frac{37}{4}y(x) = \frac{37}{2}, \qquad y(0) = 5, y'(0) = -\frac{3}{2}$$

Hint: Compute the left hand side of the differential equation with the $f(x)$ given above replacing the unknown function in the differential equation, subtract the right hand side, and see whether it simplifies to zero. (Use the Simplify[ ] command.) If it does, the differential equation is "satisfied by $f(x)$." Do the same for the two initial conditions: see if the $f(x)$ given above satisfies the two initial conditions.