Differential Equations and Linear Algebra
Math 3280
Lab #2: Introduction to Differential Equations
B. Peckham

Directions: Turn in a written lab report dealing with the tasks below. Your report should include goals, description of the procedures you used in the lab, Mathematica output with comments, and conclusions, as indicated from the “Lab Procedures and Guidelines” file. The writeup may be typed as Mathematica text, added neatly by hand to Mathematica output, or done with a word processor. Grading: Goals (G) 1, Procedures (Pro) 1, Mathematica tasks (Ma) 14 (details below), Conclusions (C) 2, Presentation and Organization (P+O) 2, Total 20.

Before starting the lab, reread the Mathematica Notes/Hints available from the link accompanying Lab 1. Do the following tasks with the help of Mathematica.

1. (8 pts) Slope field and qualitative graphical solution with varying amounts of help from Mathematica.

   (a) Sketch a slope field by hand for the differential equation \( \frac{dx}{dt} = -0.5t \), and sketch by hand the solution to the initial value problem corresponding to the initial conditions \( x(0) = 1 \).

   (b) Create the slope field for \( \frac{dx}{dt} = -0.5t \) with the following Mathematica command:
   
   ```mathematica
   VectorPlot[{1, -0.5 t}, {t, -2, 2}, {x, -2, 2}].
   ```
   Why does the vector field \( \{1, -0.5 t\} \) correspond to the slope field for the differential equation? Add the option VectorStyle → “Segment” to plot the slope marks without vector arrows.

   (c) Print out the vector field, and, by hand, sketch the solution to the differential equation corresponding to the initial condition \( x(0) = 1 \).

   (d) Use StreamPlot instead of VectorPlot to produce a graph of solutions which line up with the vector field. By hand, sketch or highlight the solution corresponding to the initial condition \( x(0) = 1 \).

   (e) Display the slope field and stream plot graphs simultaneously using the Show command. (Hint: name each plot first.)

   (f) Comment on the effort and accuracy of each of the above methods for obtaining a plot of the particular solution.

   (g) Experiment with parameters and the Manipulate command: graph a slope field for \( \frac{dx}{dt} = at \), where \( a \) is “manipulated.” Allow \( a \) to vary from at least \(-2\) to \(2\). Add another manipulated variable to manipulate the size of the plot window.

2. (3pts) Calc II Template. A template for solving Calc II type initial value problems is provided with the Mathematica Notes provided with Lab 1. Modify this template to find a formula (analytic) solution to the initial value problem:

\[
\frac{dx}{dt} = -0.5t, \quad x(0) = 1.
\]

Plot your solution. Compare it with the plot you obtained for the this same initial value problem using the slope field in part 1 of this lab.

3. (3pts) Verifying solutions to initial value problems. Consider the function:

\[
f(x) = 3e^{-x/2} \cos(3x) + 2.
\]

   (a) Plot \( f(x) \) over the interval \( 0 \leq x \leq 2\pi \). Example: Plot\([x^2, \{x,-1,1\}]\). More plotting features can be found by looking up the Plot command with the online Help.
(b) Repeat part (a) for \( f'(x) \) and \( f''(x) \).

(c) Obtain a simultaneous plot of \( f, f', \) and \( f'' \) over the interval \( 0 \leq x \leq 2\pi \). Example of Plot command: \( \text{Plot}[\{x^2, .3 \ x, 1-x\}, \{x,-1.5,1\}] \).

(d) Use Mathematica to show that \( f(x) \) satisfies the initial value problem:

\[
y''(x) + y'(x) + \frac{37}{4}y(x) = \frac{37}{2}, \quad y(0) = 5, y'(0) = -\frac{3}{2}
\]

Hint: Compute the left hand side of the differential equation with the \( f(x) \) given above replacing the unknown function in the differential equation, subtract the right hand side, and see whether it simplifies to zero. (Use the Simplify[ ] command.) If it does, the differential equation is “satisfied by \( f(x) \).” Do the same for the two initial conditions: see if the \( f(x) \) given above satisfies the two initial conditions.