Math 3280: DE+LA. Test 2 partial answers. Prof. Bruce Peckham

1. (a) Plug and check

   (b) $y(x) = c_1 e^x + c_2 e^{-3x} + (-x + 1)$

2. $y(x) = c_1 e^{-3x} + c_2 x e^{-3x}$

3. Start with $c_1 1 + c_2 x + c_3 x^2 = 0$. Show all three constants must equal zero. Several ways to do this. The two most used ways are

   (a) Use the original equation along with two more equations obtained by differentiating the starting equation. The determinant of the coefficient matrix is the "Wronskian determinant" of $\{1, x, x^2\}$.

   (b) Get three equations by chosing 3 different x values, for example $0, 1, 2$. Each x determines an equation. The three equations can be solved to show all three constants are zero.

4. (a) $y(x) = c_1 1 + c_2 x + c_3 x^2$

   (b) $y(x) = 1 + 2x + \frac{3}{2}x^2$

5. Lots of answers. Any system with two equations and 5 unknowns will usually work. If you use a system whose matrix is in row echelon form, you will guarantee that the two equations do not reduce to one equation.

6. 4

7. The x axis in the plane.

8. $\begin{bmatrix} -1 & 3 & 3 \\ 2 & 1 & 1 \\ 3 & 0 & -2 \end{bmatrix} \begin{bmatrix} r \\ s \\ t \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}$

9. $(x_1, x_2) = (-3/7, 5/7)$

10. (a) $\left\{ t \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix} \right\}$ (t is any real number))

    (b) 1 (one free variable)

11. Basis: $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ (or any two vectors along with the given vector for which the three are linearly independent)

12. (a) $A^{-1} = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$

    (b) Multiply A times A^{-1} to show you get I.

13. $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

14. True. $\begin{bmatrix}1\\3\\2\end{bmatrix} = 1\begin{bmatrix}1\\1\\0\end{bmatrix} + 2\begin{bmatrix}0\\1\\1\end{bmatrix}$ by inspection, or solve the system $c_1\begin{bmatrix}1\\1\\0\end{bmatrix} + c_2\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}1\\3\\2\end{bmatrix}$ for c_1 and c_2 using, for example, row reduction. It should turn out that $c_1 = 1$ and $c_2 = 2$.

15. You must show T is closed under vector addition and scalar multiplication:

    (a) Let $\begin{bmatrix}a\\b\end{bmatrix}$ and $\begin{bmatrix}c\\d\end{bmatrix} \in T$. This means $b = 0$ and $d = 0$. Then $\begin{bmatrix}a\\b\end{bmatrix} + \begin{bmatrix}c\\d\end{bmatrix} = \begin{bmatrix}a+c\\b+d\end{bmatrix} = \begin{bmatrix}a+c\\0\end{bmatrix} \in T$. So T is closed under vector addition.

    (b) Let $\begin{bmatrix}a\\b\end{bmatrix} \in T$. This means $b = 0$. Let $c \in \Re$. Then $c\begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}ca\\c0\end{bmatrix} = \begin{bmatrix}ca\\0\end{bmatrix} \in T$. So T is closed under scalar multiplication.

16. Since y_1 and y_2 are solutions, then $y_1'' + x^2y_1' + y_1 = 0$ and $y_2'' + x^2y_2' + y_2 = 0$. Now plug $y_1 + y_2$ into the differential equation: $(y_1+y_2)'' + x^2(y_1+y_2)' + (y_1+y_2) = (y_1''+x^2y_1'+y_1) + (y_2''+x^2y_2'+y_2) = 0 + 0 = 0$.