Math 3280: DE+LA. Practice Test 3 partial answers. Prof. Bruce Peckham

1. $y_c(x) = c_1 + c_2 x, y_p(x) = \frac{x^2}{2}$
2. $y_p(x) = \frac{1}{4}e^{2x}$
3. $y(x) = c_1 e^{4x} + c_2 e^{-4x} + c_3 x e^{-4x} + c_4 \cos(2x) + c_5 \sin(2x)$
4. $D^2 + 9$
5. $y_p(t) = Ate^{2t}$
6. $G(s) = 3\frac{1}{(s-2)^2} - \frac{e^{-4s}}{s}$
7. proof in book, Example 2, p. 577
8. $f(t) = 3 + u(t-3)(t^2-3) + u(t-4)(\cos(t) - t^2)$
9. $y(t) = 2e^{4t}$
10. $Y(s) = \frac{3s+3(s^2+16)}{(s^2+16)(s^2+s+2)}$
11. $f(t) = \frac{17}{7}e^{-4t} + \frac{4}{7}e^{3t}$
12. Let $v = y'$. Then $v' = 6v + 2y + \cos(2t)$. Vector form: $\vec{x}' = \begin{bmatrix} 0 & 1 \\ 2 & 6 \end{bmatrix} \vec{x} + \begin{bmatrix} 0 \\ \cos(2t) \end{bmatrix}$, where $\vec{x} = \begin{bmatrix} y \\ v \end{bmatrix}$.
13. (a) Need to verify: $A\vec{v} = \lambda\vec{v}$: $\begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$, and $2\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$. These are equal, so it is verified.
    (b) $\vec{x}(t) = c_1 e^{2t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 e^{-3t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$
    (c) $\vec{x}(t) = 2e^{2t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 1e^{-3t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$
14. Eigenvalues/eigenvectors: $i, \begin{bmatrix} i \\ 1 \end{bmatrix}$, or $-i, \begin{bmatrix} -i \\ 1 \end{bmatrix}$

    Solution: $\vec{x}(t) = c_1 e^{-3t} \begin{bmatrix} 4 \\ 1 \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (or equivalent)
15. (a) Any one of $(0,0)$, $(2,0)$, or $(1,1)$
    (b) Vector from $(2,1)$ is $(-2,1)$; vector from $(1,1/2)$ is $(1/2,0)$
16. (Extra Credit) In book, middle of p. 588