Math 3280
Differential Equations with Linear Algebra

Test 1
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Name____

SHOW ALL WORK.

Calculators may be used for algebra and graphing only. You may not use calculators to solve differential equations or to take integrals or derivatives.

2.__________/24

3.__________/21

4.__________/14

5.__________/16

Total_________/75

Math 3280 Sample test 1. Directions: Do all problems. Show all work. Make no mistakes.

1. Obtain the general solutions to the following differential equations. If initial conditions are given, **also** find the solution that satisfies the initial value problem. Answers need not be simplified, but integrals should be evaluated whenever you can. Implicit solutions will receive full credit only if labelled as such. Clearly indicate your final answer.
   (a) (5 pts) $y' = 3y, y(0) = 2$
   (b) (5pts) $\dfrac{dP}{ds} = 3s$
   (c) (7pts) $3\dfrac{dx}{dt} + x = e^{2t}$
   (d) (7pts) $y' = \frac{x^2}{y} + \frac{1}{xy}$. (Hint: factor; no substitution is necessary.)
2. (3 pts) Give an example of a differential equation that is first order linear, but not separable.
3. (4 pts) Assume the temperature $T(t)$ inside a building behaves according to Newton's law: $\dot{T}(t) = k(T_a - T(t))$, where T_a is the outside ("ambient") temperature. Is the constant k positive or negative? Why?
4. (7pts) Sketch the slope field for $\dot{P} = -P + t$. Include enough slope marks to allow you to sketch the solution corresponding to $P(0) = 0$. (Do not solve analytically.)
5. (7pts) Let $\phi(x)$ be the solution to the initial value problem $y' = \frac{x^2}{y}, y(1) = 5$. Approximate $\phi(1.1)$ using Euler's method with a step size of 0.05. You do not need to simplify your answer. Do not solve analytically.
6. (6pts) For what values of A is Ae^{5x}) a solution to $y'' + y = 5e^{5x}$? Justify.
7. (8pts) Sketch a phase line for $\dot{R} = (R+1)(R-1)(R-2)$. Label your axis and any equilibrium points. Using only the information from the phase line, sketch possible solutions corresponding to the initial conditions $R(0) = 0.0, 1.0, 3.0$. What is the "long term behavior"(in forward time) for the solution starting at R=0? Label your axes. (Do not solve the differential equation analytically.)
8. (8pts) Consider the differential equation
$$\frac{dy}{dx} = \frac{x^2y^2 + xy^3}{x^4}.$$
   Use the substitution $y = vx$ to replace y with v. What is the new differential equation in v? Is the new equation easier to solve than the original? If so, indicate what type the new differential equation is, but don't solve it.)
9. (8pts) A 400L tank initially holds 100 L of pure water. Saltwater flows into the tank through two separate feeds. The first flows at a rate of 1 L/min with a concentration of .005kg/L. The second feed flows at a rate of 2 L/min with a concentration of .01 kg/L. The tank is kept well stirred and flows out of the tank at a rate of 4 L/min. Let $x(t)$ denote the amount in kilograms of salt in the tank at time t. Carefully write a differential equation, with initial condition, that describes the change in amount of salt in the tank as a function of time. Label your variables. (Do **not** solve.) Over what time interval would the differential equation be valid? Explain.