Directions: Do all problems. Show all work. Make no mistakes.

1. (4 pts) Classify each differential equation as linear, separable, neither or both:
   a) $y' = y\sqrt{x}$, b) $y' = x\sqrt{y}$.

2. (4pts) If $g(x)$ is a solution to the initial value problem: $y' = y^2 - 3x, y(2) = 3$, what is $g'(2)$? (Hint: don't solve the differential equation!!)

3. Obtain the general solutions to the following differential equations. If initial conditions are given, **also** find the solution that satisfies the initial value problem. Answers need not be simplified, but integrals should be evaluated whenever you can. Implicit solutions will receive full credit only if labelled as such. Clearly indicate your final answer.

   (a) (6 pts) $y' = 3x, y(1) = 2$.

   (b) (6 pts) $\dfrac{dP}{ds} = -5P,\ P(0) = 4$.

   (c) (10 pts) $\dfrac{dx(t)}{dt} + 3t^2x(t) = e^{-t^3}$. Express your answer explicitly.

   (d) (10 pts) $y' = (x^3 + x^2 + 1)y^3$.

4. Assume the temperature $T(t)$ of a coffee cup behaves according to Newton's law:

$$\dot{T}(t) = 10(70 - T(t)),$$

   where 70 is the constant temperature of the room.

   (a) (10 pts) Make the substitution: $W(t) = 10(70 - T(t))$. Determine the new differential equation after the substitution.

   (b) Extra credit.

      i. (2 pts) Solve the new differential equation by inspection.

      ii. (2 pts) *Use the solution of the new equation* to write down the explicit general solution to the original equation.

5. (10 pts) Sketch the slope field for $y'(x) = \frac{1}{2}x$. Include enough slope marks to allow you to sketch the solution corresponding to $y(0) = 0$ and the solution corresponding to $y(0) = 1$. (Do NOT solve analytically.)

6. (10 pts) Let $\phi(x)$ be the solution to the initial value problem $y' = x - y^2, y(1) = -1$. Approximate $\phi(1.1)$ and $\phi(1.2)$ using Euler's method with a step size of 0.1. You do not need to simplify your answers. Do not solve analytically.

7. (10 pts) For what values of A and m is Ax^m a solution to $3xy' + y = 0$?

8. (10 pts) Sketch a phase line for $\dot{P} = -P(P-1)(P-3)$. Label your axis and any equilibrium points. Using only the information from the phase line, sketch possible solutions corresponding to the initial conditions $P(0) = 0.5, 1.0, 2.0$. Label your axes. What initial populations allow the population to survive (that is, the population does not approach zero) in the long run. (Do not solve the differential equation analytically.)

9. (10 pts) Assume that if unchecked, the population of mathematicians would grow at a rate proportional to the number of mathematicians alive at any given time. Assume, however, that there are two checks are operating. First, because of the interfering brain waves when too many mathematicians are too close together, mathematicians are eliminated at rate proportional to their population squared. (This is independent of the effect too many mathematicians have on the rest of humanity.) Secondly, in an effort to control the population of mathematicians the government has decided to eliminate them at a rate of 100 per year. If there are currently $10,000$ living mathematicians, write a differential equation **and** initial condition, that describe the change in population as a function of time. Label any variables and constants you use; distinguish between them; indicate whether the constants should be positive or negative. (Do **not** solve.)