Math 3280 Practice Test 2 - B. Peckham

1. Obtain the general solutions to the following differential equations.

   (a) (6 pts) $y'' + 3y' - 4y = 0$.

   (b) (6 pts) $y'' - 4y' + 4y = 0$.

2. (8 pts) Given that $y(x) = c_1 e^{2x} + c_2 e^{-2x}$ is the general solution to $y'' - 4y = 0$, and that one solution to $y'' - 4y = 8$ is $y_p(x) = -2$, find the specific solution to the nonhomogeneous differential equation which satisfies the initial conditions $y(0) = 1$ and $y'(0) = 1$.

3. Evaluate the following determinants. Show your work.

   (a) (4 pts) $\begin{vmatrix} 3 & -2 & 4 \\ 0 & -3 & -2 \\ 0 & 0 & -7 \end{vmatrix}$

   (b) (8 pts) $\begin{vmatrix} 0 & 3 & -1 & 2 \\ 1 & 0 & 3 & 1 \\ 0 & 1 & -1 & -1 \\ 3 & 2 & -2 & 0 \end{vmatrix}$

4. (4 pts) If A is a 5×5 matrix, and $\det(A) = 1$, what is $\det(2A)$? Explain briefly.

5. (4 pts) Write the vector equation $c_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} 8 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -5 \\ 2 \\ 1 \end{bmatrix}$ in the form $A\mathbf{x} = \mathbf{b}$.

6. (8 pts) Solve the following linear system USING GAUSSIAN ELIMINATION (row reduction to echelon or reduced echelon form). Leave your answers as exact fractions - not calculator approximations.

$$\begin{bmatrix} 1 & 1 \\ 6 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

7. (8 pts) Find all solutions to $\begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & 2 & 0 & -1 \\ 0 & 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. Write your answer in vector form.

8. (4 pts) Write down a basis for $W = span \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$. Justify briefly.

9. (8 pts) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

   (a) Find A^{-1} using the Gauss-Jordan (row reduction) technique. Check your answer.

10. TRUE-FALSE. Justify your answer briefly. A formal proof is not required.

    (a) (4 pts) The set of all solutions to

    $$\begin{bmatrix} 2 & 1 & 4 \\ 3 & 3 & 2 \\ 6 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

    is a vector subspace of $\Re^3$.

    (b) (4 pts) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix} \right\}$ is a basis for $\Re^3$.

11. (6 pts) Prove DIRECTLY FROM THE DEFINITION OF LINEAR INDEPENDENCE that the set $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$ is linearly independent. (You are not allowed to use shortcuts like claiming that they are independent because they are not multiples of each other.)

12. (4 pts) Show that the set of functions $\{e^x, e^{2x}, e^{3x}\}$ is linearly independent.

13. (6 pts) Prove DIRECTLY FROM THE DEFINITION OF SPAN that $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is not in the span of $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$.

14. (8 pts) Consider the following subset T of $\Re^2$. PROVE that T is a vector subspace of $\Re^3$.

    $$T = \{(x_1, x_2) : x_1 + 2x_2 = 0\}$$

15. Prove that the set of solutions to $A\mathbf{x} = \mathbf{0}$ is a vector subspace.