Math 3280: DE+LA. Practice Test 3 partial answers. Prof. Bruce Peckham

1.
$$y_c(x) = c_1 + c_2 x, y_p(x) = \frac{x^2}{2}$$

2.
$$y_p(x) = \frac{1}{4}e^{2x}$$

3.
$$y(x) = c_1 e^{4x} + c_2 e^{-4x} + c_3 x e^{-4x} + c_4 \cos(2x) + c_5 \sin(2x)$$

4.
$$D^2 + 9$$

5.
$$y_p(t) = Ate^{2t}$$

6.
$$G(s) = 3\frac{1}{(s-2)^2} - \frac{e^{-4s}}{s}$$

7. proof in book, Example 2, p. 577

8.
$$f(t) = 3 + u(t-3)(t^2-3) + u(t-4)(\cos(t)-t^2)$$

9.
$$y(t) = 2e^{4t}$$

10.
$$Y(s) = \frac{3s^2 + 3(s^2 + 16)}{(s^2 + 16)(s^2 + s + 2)}$$

11.
$$f(t) = \frac{17}{7}e^{-4t} + \frac{4}{7}e^{3t}$$

12.
$$y_p(t) = At\cos(2t) + Bt\sin(2t)$$

13. Let
$$v = y'$$
. Then $v' = 6v + 2y + \cos(2t)$. Vector form: $\vec{x}' = \begin{bmatrix} 0 & 1 \\ 2 & 6 \end{bmatrix} \vec{x} + \begin{bmatrix} 0 \\ \cos(2t) \end{bmatrix}$, where $\vec{x} = \begin{bmatrix} y \\ v \end{bmatrix}$.

14. (a) Need to verify:
$$A\vec{v} = \lambda \vec{v}$$
: $\begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$, and $2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$. These are equal, so it is verified.

(b)
$$\vec{x}(t) = c_1 e^{2t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 e^{-3t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

(c)
$$\vec{x}(t) = 2e^{2t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 1e^{-3t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

15. Eigenvalues/eigenvectors:
$$i, \begin{bmatrix} i \\ 1 \end{bmatrix}$$
, or $-i, \begin{bmatrix} -i \\ 1 \end{bmatrix}$

Solution:
$$\vec{x}(t) = c_1 e^{-3t} \begin{bmatrix} 4 \\ 1 \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$
 (or equivalent)

16. (a) Any one of
$$(0,0)$$
, $(2,0)$, or $(1,1)$

(b) Vector from
$$(2,1)$$
 is $(-2,1)$; vector from $(1,1/2)$ is $(1/2,0)$