Math 3280 Practice Test 3
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1. (5 pts) From Calculus II, you know the solution to $y' = x^2$ is $y(x) = \frac{x^3}{3} + C$. Identify in this solution y_c (the solution to the corresponding homogeneous differential equation) and y_p, the particular solution.

2. (5 pts) Find one particular solution to $y' + 3y = 2e^{4x}$. (You need not find the general solution.)

3. (5 pts) Find the general solution to the constant coefficient linear homogeneous differential equation which has the following auxillary polynomial: $(r^2+1)^2(r+1)^3$ (obtained by trying a solution of the form $y(x) = e^{rx}$).

4. (5 pts) What differential operator annihilates $5e^{3x}$? Verify your answer.

5. (5 pts) Use the fact that e^{3x} and $\cos(2x)$ are both known to be solutions to

$$y''' - 3y'' + 4y' - 12y = 0$$

to write down the general solution to the differential equation.

6. (5 pts) What is the Laplace transform of $g(t) = 3t^2 - 5e^{4t}\cos(3t)$? (You may use the tables. You need not simplify your answer.)

7. (5 pts) **Use the definition of the Laplace transform** to show that if $F(s)$ is the Laplace transform of $f(t)$, then the transform of $e^{2t}f(t)$ is $F(s-2)$.

8. (10 pts) Define $g(t) = \begin{cases} 0 & 0 \le t < 3 \\ (t-3)^2 & 3 \le t. \end{cases}$ Use step functions to write $g(t)$ as a single line formula. Find the Laplace transform of $g(t)$ either directly from the definition or from tables.

9. (7 pts) Solve using the method of Laplace transforms: $y'(t) + 5y(t) = 0,\ y(0) = 3$.

10. (8 pts) Compute the Laplace transform of the solution of the initial value problem:

$$y'' - 3y' + y = 3\cos(4t);\quad y(0) = 2, y'(0) = 0.$$

(Find only $Y(s)$, not $y(t)$.) You need not simplify your answer.

11. (8 pts) Find the inverse Laplace transform of $F(s) = \dfrac{3s-5}{s^2+2s+37}$.

12. (5 pts) Find the <u>form</u> of a particular solution to $y'' + 4y = \sin(2t)$. Do not include extraneous terms and do not evaluate the "undetermined coefficients."

13. (7 pts) Write the differential equation $y''+y'-2y = 0$. as a system of first order differential equations. Write the system in vector form.

14. (5 pts) Is $\begin{pmatrix} e^{-3t} \\ -2e^{-3t} \end{pmatrix}$ a solution to $\mathbf{x}' = \begin{pmatrix} 1 & 2 \\ 2 & -2 \end{pmatrix} \mathbf{x}$? Justify.

15. (9 pts) Let $A = \begin{pmatrix} -2 & -4 \\ -1 & 1 \end{pmatrix}$. One eigenvalue-eigenvector pair is $(-3, \begin{pmatrix} 4 \\ 1 \end{pmatrix})$. Find the other eigenvalue and a corresponding eigenvector. Use this to find the general solution to $\mathbf{x}' = A\mathbf{x}$.

16. (6 pts) Consider the system of differential equations

$$\dot{x} = -\frac{1}{2}x, \quad \dot{y} = y - y^2$$

   (a) Find all equilibrium points.

   (b) Assume the initial conditions $x(0) = 1, y(0) = 2$. Determine the velocity vector of the solution $(x(t), y(t))$ at $t = 0$.

17. Extra Credit (5pts) Solve $\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 2 & -4 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$