Math 3280: DE+LA. Test 3 Practice Problem answers. Prof. Bruce Peckham

1. $y(x) = c_1 e^{5x} + c_2 e^{-2x} - \frac{2}{5} e^{3x}$
2. $y'' - 6y' + 9y = 0$
3. Evals: 1 or -1. For 1: $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$. For -1: $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.
4. $y(x) = c_1 e^{3x} + c_2 e^{-4x} + c_3 x e^{-4x} + c_4 x^2 e^{-4x} + c_5 \cos(3x) + c_6 \sin(3x)$
5. (a) $y_p(t) = A\cos(2t) + B\sin(2t)$

   (b) $y_p(t) = Ate^t$
6. See book, proof of Theorem 1, p. 590-591.
7. $\frac{2}{s^2} - e^{-5s}(\frac{2}{s^2} + \frac{10}{s})$
8. $f(t) = e^{4t} + 2e^{-3t}$
9. $y(t) = 3e^{4t}$
10. $y(t) = 2\cos(2t) + \frac{1}{2}\sin(2t)$
11. $Y(s) = \frac{1}{s^2-3s+5}(\frac{3s}{s^2+1} + 5s^2 + 7s - 13)$
12. (a) Let $v = \dot{x}$, and $\vec{x} = \begin{pmatrix} x \\ v \end{pmatrix}$. Then $\dot{\vec{x}} = \begin{pmatrix} 0 & 1 \\ -\frac{5}{m} & -\frac{3}{m} \end{pmatrix} \vec{x} + \begin{pmatrix} 0 \\ -9.81 \end{pmatrix}$.

    (b) Up. Since $g = -9, .81$, which must be down, then positive x must be up.
13. $\vec{x} = c_1 e^{2t} \begin{pmatrix} \cos(3t) - 4\sin(3t) \\ 2\cos(3t) \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 4\cos(3t) + \sin(3t) \\ 2\sin(3t) \end{pmatrix}$
14. (a) $(0,0)$ and $(1,1)$.

    (b) Both orbits go counterclockwise. For example, at $(2,3)$, the velocity vector is $< -4, 3 >$.