Math 3280, Differential Equations with Linear Algebra

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Brief Course Summary

Differential Equations:

1. Analytic Solutions

Order	Dim	Type	Solution Technique
1	1	Simplest nontrivial: $y' = ay$ Separable (nonlinear) $y' = f(x)g(y)$ linear: $y' + p(x)y = q(x)$ not sep or linear: $y' = f(x, y)$	Exponential solutions!! Separation of Variables integrating factor Substitutions
$n \geq 1$	1	Linear const coeff homogeneous $y'' + ay' + cy = 0$ or $L[y] = 0$ .... –nonhomogeneous: add y_p $y'' + ay' + by = g(x)$ or $L[y] = g$	Try e^{rx}; 3 cases for 2nd order Laplace transforms Convert to 1st order system .... Lucky guess/undet. coeffs/Annihilators variation of parameters $(y_1, ..., y_n \to y_p)$ Laplace transforms - esp. g discontin.
$n > 1$	1	Linear nonconst coeff: $a(x), b(x)$	No general technique (but $y_1 \to y_2 = vy_1$)
$n \geq 2$	1	Nonlinear: $y^{(n)} = f(y^{(n-1)}, ..., y, x)$	No general technique
1	n	Linear const coeff homog systems $\mathbf{x}' = A\mathbf{x}$	Eigenvalues/eigenvectors; 2 cases for 2D (rep roots not covered)
1	n	Nonlinear systems: $\mathbf{x}' = f(\mathbf{x}, t)$	No general technique

2. Qualitative Solutions
   (a) 1D Automomous ($y' = f(y)$): Equilibria, phase line, vector field, direction field
   (b) 1D ANY ($y' = f(y, x)$): Slope field
   (c) 2D Automomous ($\mathbf{y}' = f(\mathbf{y})$): Equilibria, phase plane, vector field, direction field

3. Numerical Solutions
   (a) ANY!!!!: Euler's method (not covered: Runge-Kutta, ...)

4. Models - construct given verbal information ("X is proportional to Y")
   (a) Exponential growth (population), decay (radioactive decay)
   (b) Heating/Cooling
   (c) Falling object
   (d) Mixing
   (e) Logistic population growth
   (f) Spring/mass system - horizontal or vertical
   (g) Population models (predator-prey)

Linear Algebra

1. Solve $A\mathbf{x} = \mathbf{b}$ (Row reduction, echelon forms, (0, 1, ∞: free params.))

2. For $n \times n$: Det(A), A^{-1} (if Det(A) $\neq 0$)

3. Vector Space/subspace, basis, linearly independent, span, dimension

4. Linear transformation - "kernel" or "null space"
   Examples: D, integration, L (for lhs of linear differential equation), Laplace transform, multiply by matrix A, Annihilators

5. Theorems: The following are vector subspaces:
   (a) Solutions to $A\mathbf{x} = \mathbf{0}$ (Dimension depends on form of reduced matrix.)
   (b) Solutions to $L[y] = 0$ (Dimension depends on order of L.)