

Differential Equations:

1. Analytic Solutions

Order	Dim	Type	Solution Technique
1	1	Separable(nonlinear) linear not sep or linear	Separation of Variables integrating factor Substitutions
$n \geq 1$	1	Linear const coeff homogeneous -nonhomogeneous	Try e^{rx} ; 3 cases for 2nd order Laplace transforms Lucky guess/undet. coeffs/Annihilators variation of parameters ($y_1, \dots, y_n \rightarrow y_p$) Laplace transforms
$n \geq 1$	1	Linear nonconst coeff	No general technique (but $y_1 \rightarrow y_2 = vy_1$)
$n \geq 2$	1	Nonlinear	No general technique
1	n	Linear const coeff homog systems	Eigenvalues/eigenvectors; 2 cases for 2D (rep rts not covered)
1	n	Nonlinear systems	No general technique

2. Graphical Solutions

- (a) 1D Automomous: Equilibria, phase line, vector field, direction field
- (b) 1D ANY: Slope field
- (c) 2D Automomous: Equilibria, phase plane, vector field, direction field

3. Numerical Solutions

- (a) ANY!!!!: Euler's method, Runge-Kutta, ...

4. Models

- (a) Exponential growth (population), decay (radioactive decay)
- (b) Heating/Cooling
- (c) Falling object
- (d) Mixing
- (e) Logistic population growth
- (f) Spring/mass system - horizontal or vertical
- (g) Population models (predator-prey)

Linear Algebra

- 1. Solve $Ax = B$ (Row reduction, echelon forms)
- 2. For $n \times n$: $\text{Det}(A)$, A^{-1} (if $\text{Det}(A) \neq 0$)
- 3. Vector Space/subspace, basis, linearly independent, span
- 4. Linear transformation - "kernel" or "null space"
Examples: D , integration, L (for lhs of linear differential equation), Laplace transform, multiply by matrix A , Annihilators
- 5. Theorems: The following are vector subspaces:
 - (a) Solutions to $Ax = 0$
 - (b) Solutions to $L[y] = 0$