Differential Equations and Linear Algebra
Math 3280
Lab #2: Introduction to Differential Equations

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Tues. Sept. 12, 2006
Due: Wed. Sept. 20, 2006

Directions: Turn in a written lab report dealing with the tasks below. Your report should include goals, description of the procedures you used in the lab, *Mathematica* output with comments, and conclusions, as indicated on the "Lab Procedures and Guidelines" handout. The writeup may done either with a word processor or neatly by hand. Comments on *Mathematica* printouts may be either typed as text from within Mathematica, or written by hand.

Do the following tasks with the help of *Mathematica.*

1. Slope field and graphical solution with *Mathematica*.

   (a) Sketch the slope field for $\frac{dx}{dt} = -0.5t$ with the following *Mathematica* commands:
   - << Graphics`PlotField`
   - PlotVectorField[{1, -0.5 t}, {t,-2,2}, {x,-2,2}]

   The first line loads a package to set up additional graphics functions, including the plotting of "vector" fields.)

   Print out the vector field, and, BY HAND, sketch the solution to the differential equation corresponding to the initial conditions $x(0) = 1$.

2. Differential Equations tasks. Consider the function:

$$f(x) = 3e^{-x/2}\cos(3x) + 2$$

   (a) Plot $f(x)$ over the interval $0 \le x \le 2\pi$. Example: Plot[x^2, {x,-1,1}]. More plotting features can be found by looking up the Plot command with the online Help.

   (b) Repeat part (a) for $f'(x)$ and $f''(x)$.

   (c) Obtain a simultaneous plot of f, f', and f'' over the interval $0 \le x \le 2\pi$. Example of Plot command: Plot[{x^2, .3 x, 1-x}, {x,-1.5,1}].

   (d) Use Mathematica to show that $f(x)$ satisfies the initial value problem:

$$y''(x) + y'(x) + \frac{37}{4}y(x) = \frac{37}{2}, \qquad y(0) = 5, y'(0) = -\frac{3}{2}$$

   Hint: Compute the left hand side of the differential equation with the $f(x)$ given above replacing the unknown function in the differential equation, subtract the right hand side, and see whether it simplifies to zero. (Use the Simplify[ ] command.) If it does, the differential equation is "satisfied by $f(x)$." Do the same for the two initial conditions: see if the $f(x)$ given above satisfies the two initial conditions.

3. Calc II Template. A template for solving Calc II type initial value problems is provided with the Mathematica Notes part of the Lab 1 handout. Modify this template to find a formula solution to the initial value problem:

$$\frac{dx}{dt} = -0.5t, \quad x(0) = 1.$$

   Plot your solution. Compare it with the plot you obtained for the this same initial value problem using the slope field in part 1 of this lab.