Differential Equations with Linear Algebra, Math 3280
Lab #3
Graphical, Numerical, and Analytic solutions to the Logistic Differential Equation.

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Tues. Sept. 19, 2006

Due: Wed. Sept. 27, 2006

Directions: The following tasks are intended to supplement the homework problems in sections 1.3 (slope fields), 1.4 (separable D.E.s), 2.1 (population models), and 2.2 (equilibrium solutions and stability) in Edwards and Penny.

Turn in a written lab report for the following tasks. Adhere to the "Lab Procedures and Guidelines."

Consider the initial value problem:

$$\frac{dy}{dt} = y(3-y), \quad y(0) = y_0$$

Method 1: Since this differential equation is autonomous (y' depends only on y, not on t) it is appropriate to sketch a phase line. Do it (by hand). Identify any equilibrium solutions (critical points), and indicate the stability of each. Now, using only the information on your phase line, sketch graph in the (t, y) plane which *could* be solutions corresponding to initial conditions $y_0 = 1, 3.5, -0.5$.

Method 2: Sketch from direction field. Using Mathematica (see p. 29 in Edwards and Penny or Lab 2), obtain a plot of the direction field for the above differential equation. Print this plot out. By hand, sketch the solution corresponding to the three initial conditions $y_0 = 1, 3.5, -0.5$.

Method 3: Computer graph of numerical solution. Compute the solution corresponding to the initial condition $y_0 = 1$. Hint: Use variations of the NDSolve command below:
sln1 = NDSolve[{y'[t] == y[t] (3-y[t]),y, y[0]==1},{t,-5,5}]
(This computes the numerical solution to $y' = y(3-y)$ with initial condition $y(0) = 1$. It is saved (in sln1) as an interpolating function which can only be evaluated for t between -5 and 5.)
Plot[y[t]/.sln1,{t,-3,4}] (This plots the numerical solution. The plot range (-3,4) should be inside the interval on which the solution was computed (-5,5).)

Method 4: Find a formula for the general solution, then graph specific solutions. The given differential equation is separable. Find a formula for the general solution using one of the following three strategies (indicate which one you are using):

(a) Solve completely with Mathematica using the Separation of Variables Template provided.
(b) Organize by hand but do some tasks with Mathematica.
(c) Solve completely by hand.

Find the specific formula solution for the initial conditions: $y_0 = 1$. As a check on your calculations, the solutions should all be equivalent to

$$y(t) = \frac{3y_0}{y_0 + (3 - y_0)e^{-3t}}.$$

Mathematica Caution: When *Mathematica* integrates $\frac{1}{3y-y^2}$ it obtains an expression with Log's, but it is the "complex-valued" Log function, which allows negative (as well as complex) numbers as an argument. If you ignore this, you will see that once the solution is simplified, the resulting function is real (that is, no complex numbers appear). If you are unable to obtain the solution, either using Mathematica or by hand, continue on with the problem using the above formula solution.

Check your solution. Either using *Mathematica* or by hand, plug your solution (or the above formula for $y(t)$, if you weren't able to obtain a solution) back into the original differential equation to verify that it is, in fact, a solution to the initial value problem (the differential equation *and* initial condition). **Plot your analytic solutions.** Use *Mathematica* to plot the solution corresponding to the initial condition $y_0 = 1$.

Compare Methods 1 – 4. Are the graphs obtained from all four methods consistent with each other? Discuss both the accuracy and the amount of work necessary in obtaining a graph with each method. Which method allows you to most easily evaluate a) $\phi(1)$ b) $\lim_{t\to\infty} \phi(t)$, where $\phi(t)$ is the solution to the differential equation above with initial condition $y_0 = 1$?

Extra Credit: Compute the solution for $y(0) = 3.5$ using methods 3 and 4. Compare with the solutions from methods 1 and 2. Hand in on a separate sheet to Prof. Peckham