Differential Equations I
Math 3280
Lab #8: Differential Equations and Linear Algebra with Mathematica
B. Peckham
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Due: Wed. Nov. 22, 2006

Our use of *Mathematica* so far in the course has been limited to performing tasks that were learned in earlier courses: integration, differentiation, graphing. This is by design. Techniques you are learning for the first time are best learned by hand. Now that we are nearing the end of the course, it is appropriate to introduce you to some commands that perform the tasks you have been learning to do by hand. These commands include DSolve and NDSolve for differential equations, and LinearSolve, Eigenvector, Eigenvalue, NullSpace, Inverse, Transpose, and RowReduce for Linear Algebra. Syntax for individual commands are available in the *Mathematica* book or from the *Mathematica* HELP menu.

Directions: Either have your TA/instructor check off the following tasks or turn in a written lab report dealing with the tasks below. You should understand and be able to explain to your TA/instructor each output line generated by *Mathematica*. If you elect to write a report, your report should include goals, a statement of all the problems solved during the lab, *Mathematica* output with comments interpreting the output, and conclusions, as indicated on the "Lab Procedures and Guidelines" handout. The writeup may done either within *Mathematica*, with a word processor, or neatly by hand.

1. Solving differential equations analytically and numerically. Solve the following:
   (a) An analytical solution to $\frac{dx}{dt} = -.5x$. Use DSolve[x'[t]==-.5*x[t], x[t],t]. The first argument is the differential equation (or equations and initial conditions, if given, as in the next problem), the second argument is the variable we are solving for, and the last is the independent variable.
   (b) An analytical solution to $\frac{dx}{dt} = -.5x,\ x(0) = 2$. Use sln1=DSolve[{x'[t]==-.5*x[t], x[0]==2}, x[t],t] to complute the analytical solution and save it in sln1. Plot sln1 for $t \in [-1, 2]$ using Plot[x[t]/.sln1, {t,-1,2}].
   (c) A numerical solution to $\frac{dx}{dt} = -.5x,\ x(0) = 2$ for t in the interval $[-1, 2]$. Use sln2=NDSolve[{x'[t]==-.5*x[t], x[0]==2}, x[t], {t,-1,2}] to obtain the solution and save it in sln2. Use Plot[x[t]/.sln2, {t,-1,2}] to plot the result.
   (d) Use DSolve to obtain an analytical solution to $y'' + 3y' + 2y = 3e^{4t}$.

2. The logistic differential equation (again). Consider the initial value problem:

$$\frac{dy}{dt} = y(3 - y), \quad y(0) = y_0$$

   Use DSolve to find a formula (analytic) solution for each of the three initial conditions: $y_0 = 1, 3.5, -0.5$. As in Lab 2, the solutions should all be equivalent to

$$y(t) = \frac{3y_0}{y_0 + (3 - y_0)e^{-3t}}.$$

   Find and plot the graph of the analytic solution you just obtained for $y_0 = 1$ for t in the interval $[-3, 5]$. Compute and plot the same solution (with $y(0) = 1$) numerically.

3. Matrix manipulation. Let $A = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$. Find A^{-1}, A^T, $|A|$ and A^2. (Note that matrices are lists of lists. Assign A = {{1,2},{1,3}}. Use Inverse[A], Transpose[A], Det[A] and A.A, respectively. NOTE: The . is necessary when performing matrix multiplication or matrix times vector!!) Multiply A times its inverse. Do you get what you expect?

4. Solving Linear Systems. Solve $Ax = b$ for x using LinearSolve[A,b].

   (a) $A = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. (Assign A = {{1,2},{1,3}} and b={1, 0}. Then solve using LinearSolve[A,b].)

   (b) $A = \begin{pmatrix} 3 & 5 & -1 \\ 1 & 2 & 1 \\ 2 & 5 & 6 \end{pmatrix}$ and $b = \begin{pmatrix} 14 \\ 3 \\ 2 \end{pmatrix}$. Why might *Mathematica* have trouble here?

   (c) Repeat the previous problem using RowReduce[B], where B is the augmented 3×4 matrix. Use the reduced matrix to determine (by hand) the solution.

5. Nullspace. Find the nullspace (the solutions to $Ax = 0$) where A is the 3×3 matrix above. (Use Nullspace[A] – a basis for the nullspace is returned.)

6. Eigenvalues and Eigenvectors. Use Eigenvalues[m] and Eigenvectors[m] to determine the eigenvalues and eigenvectors of the following matrices:

   $\begin{pmatrix} 3 & -1 \\ -5 & -1 \end{pmatrix}$, $\begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}$, and the 3×3 matrix above.

   For any one of the matrices, multiply the matrix times any one of its eigenvectors, and compare this to the corresponding eigenvalue times the eigenvector.