Math 3280
Differential Equations with Linear Algebra Practice Test 2
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1. Obtain the general solutions to the following differential equations. If initial conditions are given, **also** find the solution that satisfies the initial value problem
   (a) $y'' + 2y' + 3y = 0, y(0) = 2, y'(0) = 0.$
   (b) $y'' + 2y' + y = 5e^{3x}$. Hint: Try $y_p(x) = Ae^{3x}$.

2. Let $A = \begin{pmatrix} 0 & 1 & 3 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 3 & 1 & -5 \\ 2 & -1 & 2 \\ 0 & 1 & 3 \end{pmatrix}$. Show your work to compute (4 pts) $AB - 2I + 3A$

3. Evaluate the following determinants. Show your work.
   (a) $\begin{vmatrix} 3 & 1 \\ 6 & 2 \end{vmatrix}$
   (b) $\begin{vmatrix} 1 & -1 & 3 \\ 2 & 0 & 1 \\ 3 & 1 & 4 \end{vmatrix}$
   (c) $\begin{vmatrix} 1 & 3 & -1 & 2 \\ 2 & 4 & 3 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 2 & -2 & 2 \end{vmatrix}$

4. Solve the following linear system USING GAUSSIAN ELIMINATION: $\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

5. Find a basis for the space of solutions to $\begin{pmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \\ 4 & 3 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

6. Assume you are trying to solve the linear system $A\mathbf{x} = \mathbf{b}$ where A is a 3×3 matrix. Assume the augmented matrix has been row reduced to row echelon form.
   (a) Write down a form for which you could conclude that the original system had no solutions.
   (b) Is it easier to determine the solution to the system of equations if the augmented matrix has been reduced to row echelon form (REF) or reduced row echelon form (RREF). Explain briefly.

7. TRUE-FALSE. The following sets of vectors are bases for $\Re^3$. Justify your answer briefly. A formal proof is not required.

(a) $\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} \right\}$

(b) $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \right\}$

8. Let $A = \begin{pmatrix} 3 & 1 \\ 6 & 1 \end{pmatrix}$. Find A^{-1} using the Gauss-Jordan (row reduction) technique. Check your answer.

9. Let $\Re^2 = \{\mathbf{x} = (x_1, x_2) : x_1 \in \Re, x_2 \in \Re\}$.

   (a) Let $S = \{\mathbf{x} = (x_1, x_2) \in \Re^2 : x_1 = 0\}$. Is S a vector subspace of $\Re^2$? Justify briefly.

   (b) Let $T = \{\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \Re^2 : A\mathbf{x} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}\}$. Is T a vector subspace of $\Re^2$? Justify briefly.

10. Write down a basis for $\mathcal{P}_2 \equiv \{a_0 + a_1 x + a_2 x^2 : a_i \in \Re\}$.

11. Write down the system of equations needs to be solved in order to show that the set of vectors $\left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \right\}$ spans $\Re^3$. DO NOT SOLVE the system, but discuss what properties of the solution would determine whether the three vectors span $\Re^3$.

12. Prove that the set of solutions to $A\mathbf{x} = \mathbf{0}$ is a subspace of $\Re^n$ (A is an $m \times n$ matrix.)