Math 3280, Differential Equations with Linear Algebra
Sample Test 3 B. Peckham

1. (4pts) From Calculus II, you know the solution to $y' = x$ is $y(x) = \frac{x^2}{2} + C$. Identify in this solution y_h (the solution to the corresponding homogeneous differential equation) and y_p, the particular solution.

2. (4pts) Write a second order, linear, constant coefficient, homogeneous differential equation that has both e^{-x} and $4e^{-8x}$ as solutions.

3. (5pts) Compute the Laplace transform of e^{3t} directly from the definition.

4. (4pts) What is the Laplace transform of $g(t) = 3t^2 - e^{3t}\cos(4t)$? (You may use the tables.)

5. (5pts) Find one particular solution to $y' + 3y = 4e^{2x}$.

6. (5pts) Find the general solution to the constant coefficient linear homogeneous differential equation which has the following auxillary polynomial: $(r^2-1)(r^2+1)^2(r+1)^3$ (obtained by trying a solution of the form $y(x) = e^{rx}$).

7. (6pts) Use the formula for the transform of $tf(t)$ to determine the Laplace transform of $t\cos(3t)$? You need not simplify your answer.

8. (8pts) Define $f(t) = \begin{cases} 0 & 0 \le t \le 2 \\ (t-2)^2 & t > 2. \end{cases}$ Use step functions to write $f(t)$ as a single line formula and find the Laplace transform of $f(t)$.

9. (7pts) Solve using the method of Laplace transforms: $y(t) = -5y'(t),\ y(0) = 3$.

10. (7pts) Find the inverse Laplace transform of $F(s) = \dfrac{3s-5}{s^2+2s+37}$.

11. (6pts) Show how to obtain the formula for the Laplace transform of f'' from the formula for the Laplace transform of f'. (Do not use directly either the formula for the transform of $f''(t)$ or the transform of $f^{(n)}(t)$.

12. (8pts) Compute the Laplace transform of the solution of the initial value problem: $y'' - 3y' + y = 3\cos(2t);\ \ y(0) = 0, y'(0) = -2$. (Find only $Y(s)$, not $y(t)$.)

13. (8pts) Find the <u>form</u> of a particular solution to the following differential equations:

    (a) $y'' + 4y = \sin(t)$
    (b) $y'' + 4y = \sin(2t)$

    Do not include extraneous terms and do not evaluate the "undetermined coefficients."

14. (8pts) Given that e^{3x} and $\cos(2x)$ are solutions to $y''' - 3y'' + 4y' - 12y = 0$, explain (briefly) how you would determine the general solution to the differential equation $y''' - 3y'' + 4y' - 12y = x^{-1}e^x$. If you remember the names of any techniques you would use, include them, but don't carry out any of the algebra of finding the solution. Include in your answer the form of the general solution.

15. (5 pts) Write the differential equation $y''' + 3y'' + y' - 2y = 0$. as a system of first order differential equations. Write the system in vector form.

16. (4 pts) Is $\begin{pmatrix} e^{-3t} \\ -2e^{-3t} \end{pmatrix}$ a solution to $\mathbf{x}' = \begin{pmatrix} 1 & 2 \\ 2 & -2 \end{pmatrix} \mathbf{x}$?

17. (9 pts) Let $A = \begin{pmatrix} -2 & -7 \\ -1 & 4 \end{pmatrix}$. One eigenvalue-eigenvector pair is $(-3, \begin{pmatrix} 7 \\ 1 \end{pmatrix})$. Find the other eigenvalue and a corresponding eigenvector. Use this to find the general solution to $\mathbf{x}' = A\mathbf{x}$.

18. Consider the system of differential equations

$$\dot{x} = -\frac{1}{2}x, \quad \dot{y} = y - y^2$$

   (a) Find all equilibrium points.

   (b) Below is the the orbit of the solution to the above differential equation which has initial conditions $(x(0) = 1, y(0) = 2$. Determine the velocity vector of the solution at time zero, and plot it on the phase plane diagram. Is the vector consistent with the numericallly drawn orbit? Why? Describe the long term fate of the solution whose orbit is drawn.