1. Find the general solution to $y'' - 3y' - 10y = 4e^{3x}$. Do not use Laplace transforms.

2. Write a linear, constant coefficient, homogeneous differential equation that has $5xe^{3x}$ as one solution.

3. Compute the eigenvalues and any one eigenvector for the matrix $A = \begin{pmatrix} 2 & -3 \\ 1 & -2 \end{pmatrix}$.

4. Find the general solution to the constant coefficient linear homogeneous differential equation which has the following auxiliary polynomial: $(r-3)(r+4)^3(r^2+9)$.

5. Write down the **form** of the guess for a particuar solution for the following differential equations. You need not evaluate the undetermined coefficients in your guess.

   (a) $y'' - 3y' + 2y = 3\cos(2t)$
   (b) $y'' - 3y' + 2y = 3e^t$

6. **Use the definition of the Laplace transform** to show that if $F(s)$ is the Laplace transform of $f(t)$, then $F(s-3)$ is the transform of $e^{3t}f(t)$.

7. **Use the definition of the Laplace transform** to compute the Laplace transform of $g(t)$ where $g(t) = \begin{cases} 2t & 0 \le t < 5 \\ 0 & t \ge 5. \end{cases}$ Write down a "single-line formula" for $g(t)$.

8. Find the inverse Laplace transform of $F(s) = \dfrac{3s-5}{s^2 - s - 12}$.

9. Solve completely using the method of Laplace transforms:
$$y'(t) = 4y(t), \quad y(0) = 3$$

10. Solve completely using the method of Laplace transforms:
$$y'' + 4y = 0, \quad y(0) = 2, y'(0) = 1$$

11. Compute the Laplace transform of the solution of the initial value problem below. (Find only $Y(s)$, not $y(t)$. You do not need to simplify your answer.)
$$y''' - 3y' + 5y = 3\cos(t); \quad y(0) = 5, y'(0) = 7, y''(0) = 2$$

12. A spring-mass system oscillating vertically is modeled by the differential equation $m\ddot{x} = m(-9.81) - 3\dot{x} - 5x$, where $x(t)$ is the displacement of the spring.

    (a) Write this second order differential equation as an equivalent system of first order differential equations. Write the system in vector form.
    (b) Is the positive direction for x up or down? How do you know?

13. Assume that $2+3i$ is an eigenvalue for a 2×2 matrix A and that $\begin{pmatrix} 1+4i \\ 2 \end{pmatrix}$ is a corresponding eigenvector. Write down the general solution to $\dot{\mathbf{x}} = A\mathbf{x}$.

14. Consider the following "predator-prey" system of differential equations:
$$\dot{x} = x - xy, \quad \dot{y} = -y + xy$$

    (a) Find all equilibrium points.
    (b) The following is a phase portrait showing three orbits, generated numerically, in the phase plane for the above system. Put arrows on the two closed curves to indicate which way the orbit proceeds as time increases.