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Real Variables, Math 4326

Quiz 1, Spring 2016

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(5 pts) Show that the following subset W of $\Re^3$ is a vector subspace of $\Re^3$. (You need not show the properties that are "inherited" from $\Re^3$. That is, show only $\vec{0} \in W$ and that W is closed under vector addition and scalar multiplication.)

$$W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \Re^3 : x_1 + x_2 = 0, \text{and } x_3 = 0. \right\}$$

i Let $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \vec{0}$. then $v_1 + v_2 = 0+0 = 0$ and $v_3 = 0$ so $\vec{0} \in W$. //

ii Let $\vec{v}, \vec{u} \in W$. $\Rightarrow$ $\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$, $\vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$ / where $u_1 + u_2 = 0$, $u_3 = 0$
$v_1 + v_2 = 0$, $v_3 = 0$

Let $\vec{c} = \vec{u} + \vec{v} \Rightarrow$ $\vec{u} + \vec{v} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \\ u_3 + v_3 \end{bmatrix}$

$c_1 + c_2 = (u_1 + v_1) + (u_2 + v_2) = (u_1 + u_2) + (v_1 + v_2) = 0 + 0 = 0.$

$c_3 = u_3 + v_3 = 0 + 0 = 0.$

$\therefore \vec{u} + \vec{v} \in W.$ //

iii Let $\vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \in W$, $r \in \mathbb{R}$. $\vec{u} \in W \Rightarrow u_1 + u_2 = 0$, $u_3 = 0$.

Let $\vec{c} = r\vec{u}$. That is, $r\vec{u} = \begin{bmatrix} r u_1 \\ r u_2 \\ r u_3 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$, So, $c_1 + c_2 = r u_1 + r u_2$
$= r(u_1 + u_2) = r \cdot 0 = 0$

$r u_3 = r \cdot 0 = 0$

$\therefore r\vec{u} \in W.$ //

i, ii + iii $\Rightarrow$ W is a vector Subspace of $\mathbb{R}^3$ //