

2. (7pts) For each of the following sets A below, fill in the following table. Put Y(es) or N(o) in the “?” columns according to whether the sets have the indicated properties. No justification required. E stands for Euclidean metric; d stands for discrete metric; \mathbb{R} is the real numbers Q is the rational numbers.

Space	A	\overline{A} (closure)	A° (interior)	Open?	Closed?	Compact?	Connected?
\mathbb{R}, E	Q						
\mathbb{R}, E	K (Cantor set)						
$[0, 5), E$	$[0, 2)$						
\mathbb{R}, d	Q						
\mathbb{R}, E	$\{\frac{1}{n}\}_{n=1}^\infty$						

3. (3pts) Explain how the theorem that “continuous functions map connected sets to connected sets” is used to prove the Intermediate Value Theorem in Calculus.

4. (3 pts) Explain why any one point set is closed in any metric space.

Part II. Do any 3 of the following 4 proofs. 8 points each. 24 points total.

1. Prove that “ U open in the range implies $f^{-1}(U)$ is open in the domain of f ” implies the ϵ - δ definition for continuity at any point in the domain of f .

2. Show that if A and B are both open sets in a metric space M , then $A \cap B$ is open in M .

3. Show that if $f : M_1 \rightarrow M_2$ is continuous, and M_1 is connected, then $f(M_1)$ is connected.

4. Show that $A \subset M$ is open, then its complement, A' , is closed in M .