1. Definitions

(a) Infinite series
(b) Convergence vs divergence of a series
(c) Conditional convergence vs absolute convergence (using partial sums)
(d) Rearrangement of a series.
(e) Cantor set
(f) Binary, ternary expansions
(g) Homeomorphism (1-1, onto, continuous, continuous inverse)
(h) Subspace topology
(i) Diameter of a set: \( \text{diam}(A) \).
(j) Connected, disconnected, disconnection of a set (space), path connected
(k) Derivative of \( f \) at a point.
(l) Riemann integral: \( \int_a^b f(x)dx = \lim_{||P||\to 0} R(f, P, T) \).
(m) Darboux integral: \( T = \sup P L(f, P), I = \inf P U(f, P), I = T = L \).

2. Proofs to know:

(a) Geometric series: Prove that \( \sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \) for \( |x| < 1 \).
(b) \( p \)-series.
   i. Prove that \( \sum_{n=1}^{\infty} \frac{1}{n} \) (show partial sums are not bounded by using the integral test).
   ii. Prove that \( \sum_{n=1}^{\infty} \frac{1}{n^2} \) converges using the integral test.
   iii. Know that \( \sum_{n=1}^{\infty} \frac{1}{n^p} \) converges iff \( p > 1 \).
(c) Prove the the ratio test: \( \lim \sup_{n \to \infty} \frac{|a_{n+1}|}{|a_n|} < 1 \) implies absolute convergence.
(d) Prove the the root test in the special case that the \( \lim \sup_{n \to \infty} |a_n|^{1/n} < 1 \) implies convergence.
(e) Prove the basic comparison test in the special case that all terms in both series are nonnegative. However, treat the case that the comparison is valid only from \( n \) beyond some \( N \).
(f) Prove that continuous maps take connected sets to connected sets.

(g) Prove that continuous maps take covering compact sets to covering compact sets.

(h) Prove that \( f : M \rightarrow N \) a homeomorphism implies that \( f : M \setminus \{m\} \rightarrow N \setminus \{f(m)\} \) is a homeomorphism.

(i) Prove path connected implies connected.

(j) Prove covering compact implies sequentially compact.

(k) Prove the Cantor middle thirds set is uncountable.

(l) Prove the Calculus I Max-min theorem

(m) Prove the Calculus I Intermediate value theorem.

(n) FTC I

(o) FTC II

(p) Rolle’s theorem

(q) MVT

(r) Product rule

(s) l’Hôpital’s rule in the east case where \( f, g, f', g' \) are all continuous at \( a \) (0/0 case only)

3. Miscellaneous results and techniques to know:

(a) Convergence/divergence tests. Know how to apply all.
   i. Alternating series test.
   ii. Basic comparison test.
   iii. Limit comparison test.
   iv. Ratio test using \( \text{lim sup} \) or \( \text{lim inf} \).
   v. Root test using \( \text{lim sup} \).

(b) Characterization of \( \sum b_n, \sum c_n \) when \( \sum a_n \) is absolutely convergent, conditionally convergent, or divergent. \( \{b_n\} \) is the subsequence of nonnegative terms in \( \{a_n\} \), \( \{b_n\} \) is the subsequence of negative terms in \( \{a_n\} \).

(c) Decide whether given series converge or diverge.

(d) Characterization in \( \mathbb{R}^1 \) of all connected sets, compact connected sets, open connected sets.

(e) Examples of sets that are connected but not path connected.

(f) Identify (non) homeomorphic sets

4. Easy HW problems.

5. Anything else we’ve covered that I think is easy.