

MIDTERM 2 TOPIC LIST: For midterm Wed. Dec. 3, 2008
(as of November 26, 2008)
Real Variables, Math 5201
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1. Definitions

- (a) Infinite series
- (b) Convergence vs divergence of a series
- (c) Conditional convergence vs absolute convergence (using partial sums)
- (d) Rearrangement of a series.
- (e) Cantor set
- (f) Binary, ternary expansions
- (g) homeomorphism (1-1, onto, continuous, continuous inverse)
- (h) Subspace topology
- (i) Diameter of a set: $\text{diam}(A)$.
- (j) Connected, disconnected, disconnection of a set (space), path connected
- (k) Derivative of f at a point.
- (l) Riemann integral: $\int_a^b f(x)dx = \lim_{\|P\| \rightarrow 0} R(f, P, T)$.
- (m) Darboux integral: $\bar{I} = \sup_P L(f, P)$, $\underline{I} = \inf_P U(f, P)$, $I = \bar{I} = \underline{I}$.

2. Proofs to know:

- (a) Geometric series: Prove that $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ for $|x| < 1$.
- (b) p -series.
 - i. Prove that $\sum_{n=1}^{\infty} \frac{1}{n}$ (show partial sums are not bounded by using the integral test.
 - ii. Prove that $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges using the integral test.
 - iii. know that $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges iff $p > 1$.
- (c) Prove the the ratio test: $\limsup_{n \rightarrow \infty} \frac{|a_{n+1}|}{|a_n|} < 1$ implies absolute convergence.
- (d) Prove the the root test in the special case that the $\limsup_{n \rightarrow \infty} |a_n|^{1/n} < 1$ implies convergence.
- (e) Prove the basic comparison test in the special case that all terms in both series are nonnegative. However, treat the case that the comparison is valid only from n beyond some N .

- (f) Prove that continuous maps take connected sets to connected sets.
 - (g) Prove that continuous maps take covering compact sets to covering compact sets.
 - (h) Prove that $f : M \rightarrow N$ a homeomorphism implies that $f : M \setminus \{m\} \rightarrow N \setminus \{f(m)\}$ is a homeomorphism.
 - (i) Prove path connected implies connected.
 - (j) Prove covering compact implies sequentially compact.
 - (k) Prove the Cantor middle thirds set is uncountable.
 - (l) Prove the Calculus I Max-min theorem
 - (m) Prove the Calculus I Intermediate value theorem.
 - (n) FTC I
 - (o) FTC II
 - (p) Rolle's theorem
 - (q) MVT
 - (r) Product rule
 - (s) l'Hopital's rule in the east case where f, g, f', g' are all continuous at a (0/0 case only)
3. Miscellaneous results and techniques to know:
- (a) Convergence/divergence tests. Know how to apply all.
 - i. Alternating series test.
 - ii. Basic comparison test.
 - iii. Limit comparison test.
 - iv. Ratio test using limsup or liminf.
 - v. Root test using limsup.
 - (b) Characterization of $\sum b_n, \sum c_n$ when $\sum a_n$ is absolutely convergent, conditionally convergent, or divergent. ($\{b_n\}$ is the subsequence of nonnegative terms in $\{a_n\}$, $\{c_n\}$ is the subsequence of negative terms in $\{a_n\}$).
 - (c) Decide whether given series converge or diverge.
 - (d) Characterization in \mathbb{R}^1 of all connected sets, compact connected sets, open connected sets.
 - (e) Examples of sets that are connected but not path connected.
 - (f) Identify (non) homeomorphic sets
4. Easy HW problems.
5. Anything else we've covered that I think is easy.