In general, the midterm will cover any topics we covered in Chapters 1-12. The focus will be on “basic” material. Homework type questions from previous assignments will be emphasized. I will attempt to make the problems noncomputationally intensive. The following list of topics should give you a more specific idea of what kinds of questions will be asked.

1. Definitions to know:

- Fixed point, periodic point, orbit, cycle, period, prime or least period
- Attracting, superattracting, repelling, neutral periodic point (orbit, cycle); use of chain rule in determining these adjectives
- Phase portrait
- Graphical analysis
- Discrete dynamical system vs. continuous dynamical system
- Bifurcation, incl. esp. saddle-node and period doubling (nondegeneracy conditions not necessary to memorize)
- The sequence space $\Sigma$, the “usual” metric on $\Sigma$, the shift map $\sigma$.
- Three properties of a chaotic system and all terminology used in the def. of the properties.
- $A$ dense in $B$ for $A \subset B$.
- Homeomorphism, topological conjugacy, topological semiconjugacy

2. Results to know:

- Relationship between the shift map on the symbol sequence space and the quadratic map (for small enough $c$) on the invariant Cantor set.
- Sarkovskii’s theorem, including Sarkovskii’s ordering
- “Negative Schwarzian Derivative Property” (which is true for any quadratic): Any attracting periodic orbit must attract a critical point.
3. Techniques to know:

- Locating fixed and periodic pts/orbits analytically (for individual maps and for families of maps)
- Locating period-$n$ pts/orbits of $f$ from graphs of $f$ and $f^n$.
- Interpreting orbit diagrams (identifying, for example, parameter values corresponding to maps with attracting orbits of a certain period, or saddle-node bifurcations or period-doubling bifurcations)
- Determining stability of periodic orbits either analytically or from graphs.
- Constructing a graph of $f$ so that $f$ has a point $p$ that is, for example, a period-$n$ point, and the derivative $(f^n)'(p)$ has a specified value. (Hint: use the chain rule!)
- Constructing the graph of iterates of $f$ given the graph of $f$.
- The construction of the invariant Cantor set $\Lambda$ for $x^2 + c$ with $c$ small enough.
- The construction of an itinerary map.
- Given a map and a change of variables, find the equation for the map in the new variables.
- Recognizing a saddle-node and/or period doubling bifurcation from a sequence of graphs of a family of maps as a parameter changes
- Determining the number of prime periodic orbits of each period for $x^2 - 2$ (equivalently $2x \pmod{1}$, $4x(1 - x)$, $\sigma$)
- Determining the number of period-$n$ windows for each $n$ in the orbit diagram for the family $x^2 + c$.
- Locating each period-$n$ window in the orbit diagram for the family $x^2 + c$.

4. Proofs to know:

- $f$ continuous, $I$ closed interval, $f(I) \subseteq I$ or $f(I) \supseteq I$ implies there is a fixed point for $f$ in $I$.
- $f$ continuous, $f$ has a periodic point implies $f$ has a fixed point
- Prove $\sigma : \Sigma \to \Sigma$ is chaotic. (Prove any or all 3 properties.)

5. Anything else we’ve covered that I think is easy.