In general, the midterm will cover any topics we covered in Chapters 1-12. The focus will be on “basic” material. Homework type questions from previous assignments will be emphasized. I will attempt to make the problems noncomputationally intensive. The following list of topics should give you a more specific idea of what kinds of questions will be asked.

1. Definitions to know:
   - Fixed point, periodic point, orbit, cycle, period, prime or least period
   - Attracting, superattracting, repelling, neutral periodic point (orbit, cycle); use of chain rule in determining these adjectives
   - Phase portrait
   - Graphical analysis
   - Discrete dynamical system vs. continuous dynamical system
   - Bifurcation, incl. esp. saddle-node and period doubling (nondegeneracy conditions not necessary to memorize)
   - The sequence space $\Sigma$, the “usual” metric on $\Sigma$, the shift map $\sigma$.
   - Three properties of a chaotic system and all terminology used in the def. of the properties.
   - A dense in $B$ for $A \subset B$.
   - Homeomorphism, topological conjugacy, topological semiconjugacy

2. Results to know:
   - Relationship between the shift map on the symbol sequence space and the quadratic map (for small enough $c$) on the invariant Cantor set.
   - Sarkovkii’s theorem, including Sarkovskii’s ordering
   - “Negative Schwarzian Derivative Property” (which is true for any quadratic): Any attracting periodic orbit must attract a critical point.
3. Techniques to know:

- Locating fixed and periodic pts/orbits analytically (for individual maps and for families of maps)
- Locating period-n pts/orbits of f from graphs of f and fⁿ.
- Interpreting orbit diagrams (identifying, for example, parameter values corresponding to maps with attracting orbits of a certain period, or saddle-node bifurcations or period-doubling bifurcations)
- Determining stability of periodic orbits either analytically or from graphs.
- Constructing a graph of f so that f has a point p that is, for example, a period-n point, and the derivative \((f^n)'(p)\) has a specified value. (Hint: use the chain rule!)
- Constructing the graph of iterates of f given the graph of f.
- The construction of the invariant Cantor set \(\Lambda\) for \(x^2 + c\) with c small enough.
- The construction of an itinerary map.
- Given a map and a change of variables, find the equation for the map in the new variables.
- Recognizing a saddle-node and/or period doubling bifurcation from a sequence of graphs of a family of maps as a parameter changes
- Determining the number of prime periodic orbits of each period for \(x^2 - 2\) (equivalently \(2x \pmod{1}\), \(4x(1 - x)\), \(\sigma\))
- Determining the number of period-n windows for each n in the orbit diagram for the family \(x^2 + c\).
- Locating each period-n window in the orbit diagram for the family \(x^2 + c\).

4. Proofs to know:

- \(f\) continuous, \(I\) closed interval, \(f(I) \subseteq I\) or \(f(I) \supseteq I\) implies there is a fixed point for \(f\) in \(I\).
- \(f\) continuous, \(f\) has a periodic point implies \(f\) has a fixed point
- Prove \(\sigma : \Sigma \to \Sigma\) is chaotic. (Prove any or all 3 properties.)

5. Anything else we’ve covered that I think is easy.