

# Dynamical Systems

Math 5260

Lab Tasks:

Tangent (saddle-node) and period-doubling Bifurcations  
for  $x^2 + c$

Bruce Peckham

See Course homepage for Due Date.

As  $c$  decreases in the quadratic family  $x^2 + c$ , many periodic orbits are “born” through either saddle-node bifurcations or period doubling bifurcations.

Do the following for period- $q$  orbits for each of  $q = 1, 2, \dots, 4$ :

By plotting the  $q$ th iterate of the quadratic family maps  $x^2 + c$  for  $c$  values decreasing from 1 to -2, determine approximate parameter values where fixed points for the  $q$ th iterate are born. Determine whether the births are due to a saddle-node or a period-doubling bifurcation. Determine the least period of the orbit being born.

Suggestion: Use Nonlinear Web (BU Website) and/or Mathematica. Nonlinear Web has the big advantage of being already written, and having a slider to change the value of  $c$ , so you can quickly get an idea of where the bifurcations are, but it is hard to identify the bifurcation parameter values with much accuracy. *Mathematica* has the advantages of being able to plot multiple graphs at once, specifying parameter value with high precision, and specifying plot bounds to zoom in on a particular region. High precision parameter specification, and zooming in on the graphs enables a more accurate estimate of the bifurcation parameters. Multiple graphs allow us, for example, to plot both the first and second iterates to see which fixed points of the second iterate are least period 1 and which are least period 2.

(Try `Plot[{f[x,c], f[f[x,c],c]}, {x,-2,2}]` where  $f[x,c] := x^2 + c$  and  $c$  is a fixed parameter value.)

You can avoid using Nonlinear Web altogether by using the `Manipulate` command to create a slider to change the parameter  $c$ !

You might also be interested in the demo plotting the first and second iterates of a quadratic family (but for  $ax(1-x)$  instead of  $x^2 + c$ ) as a parameter changes: follow from the course homepage: ‘Prof Luis Alsedá ... software’ to ‘Movie of the graphs of the first and second iterates in a period-doubling bifurcation’.