

MIDTERM TOPIC LIST

Dynamical Systems

Math 5260

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For midterm on Monday, Oct. 9, 2017.

In general, the midterm will cover any topics we covered in Chapters 1-12. The focus will be on “basic” material. Homework type questions from previous assignments will be emphasized. I will attempt to make the problems noncomputationally intensive. The following list of topics should give you a more specific idea of what kinds of questions will be asked.

1. Definitions to know:

- Fixed point, periodic point, orbit, cycle, period, prime or least period
- Attracting, superattracting, repelling, (linearly) neutral periodic point (orbit, cycle), weakly attracting, weakly repelling; use of chain rule in determining these adjectives
- Phase portrait
- Graphical analysis
- Discrete dynamical system vs. continuous dynamical system
- Bifurcation, incl. esp. saddle-node and period doubling (nondegeneracy conditions not necessary to memorize)
- The sequence space Σ , the “usual” metric on Σ , the shift map σ .
- Three properties of a chaotic system and all terminology used in the def. of the properties.
- A dense in B for $A \subset B$.
- Homeomorphism, topological conjugacy, topological semiconjugacy

2. Results to know:

- Relationship between the shift map on the symbol sequence space and the quadratic map on the invariant Cantor set (for $c < -2$).
- Sarkovskii’s theorem, including Sarkovskii’s ordering
- “Negative Schwarzian Derivative Property” (which is true for any quadratic): Any attracting periodic orbit must attract a critical point.

3. Techniques to know:

- Locating fixed and periodic pts/orbits analytically (for individual maps and for families of maps)
- Locating period- n pts/orbits of f from graphs of f and f^n .
- Interpreting orbit diagrams (identifying, for example, parameter values corresponding to maps with attracting orbits of a certain period, or saddle-node bifurcations or period-doubling bifurcations)
- Determining stability of periodic orbits either analytically or from graphs.
- Constructing a graph of f so that f has a point p that is, for example, a period- n point, and the derivative $(f^n)'(p)$ has a specified value. (Hint: use the chain rule!)
- Constructing the graph of iterates of f given the graph of f .
- The construction of the invariant Cantor set Λ for $x^2 + c$ with $c < -2$.
- The construction of an itinerary map.
- Given a map and a change of variables, find the equation for the map in the new variables.
- Recognizing a saddle-node and/or period doubling bifurcation from a sequence of graphs of a family of maps as a parameter changes
- Determining the number of prime periodic orbits of each period for $x^2 - 2$ (equivalently $2x \pmod{1}$, $4x(1-x)$, σ)
- Determining the number of period- n windows for each n in the orbit diagram for the family $x^2 + c$.
- Locating each period- n window in the orbit diagram for the family $x^2 + c$.

4. Proofs to know:

- Prove that if f is continuous, f has a periodic point implies f has a fixed point.
- Prove either direction of the Proximity Theorem.
- Prove $\sigma : \Sigma \rightarrow \Sigma$ is chaotic. (Prove any or all 3 properties.)

5. Anything else we've covered that I think is easy.