

Real Analysis, Math 8201
List of Possible Problems for Midterm Quiz
(Fri. March 27, 2009)
as of March 23, 2009
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Be able to state and apply the following definitions:

1. Construction of Lebesgue measure, including all the definitions of the measures of special rectangles, special polygons, open sets, compact sets, outer measure, inner measure, measurable sets with finite outer measure, measurable sets.
2. Lebesgue measurable sets in terms of the outer and inner measures of the set. Be sure to include the case when the outer measure is infinite.
3. A set that is not Lebesgue-measurable. Be able to write down such a set. (You are not required to prove that it is not measurable.)
4. Fat Cantor sets
5. The Cantor-Lebesgue function.
6. An algebra and a σ -algebra.
7. The Borel σ -algebra. Know the “difference” between a set that is Lebesgue measurable, and one that is Borel measurable.
8. A null set.
9. A Lebesgue-measurable function – five equivalent definitions.
10. A simple function.

Be able to state the following results. Proofs of those with a ‘p’ should also be known. The lists of ‘properties’ below need not be memorized per se, but would be more likely to appear as true-false questions. Properties to be proved will be explicitly stated.

1. Properties $O1^p, O2^p, O3^p, O4^p, O5, O6^p$
2. Properties $C1^p, C2^p, C3^p, C4$
3. Properties $*1^p, *2^p, *3^p, *4^p, *5$
4. Theorem on Approximation (for sets in L_0 p. 45) and its Corollary (p. 46)
5. Theorem on Countable Additivity (for sets in L_0 p. 47)
6. Properties M1 – M4, M5, M6 – M10
7. $\lambda(z + tI) = t^n \lambda(I), \lambda^*(TA) = |\det T| \lambda^*(A), \lambda^*(z + A) = \lambda^*(A), \lambda^*(TA) = t^n \lambda^*(A),$
...

Simple proofs that were part of previously done homework problems, or a step in such a proof, or a similar step or proof might also be asked. In particular,

- Ch 1: 2g, 10, 18abc, 19, 43, 44 (not originally assigned), also A bdd, f cts, f(A) not bdd. Compare with K compact, f cts, f(K) compact.
- Ch 2: 3, 12, 24 (or pf of Corollary), 26, 31
- Ch 5: 2, 6, 16ab, 21