Real Analysis, Math 8201

List of Possible Problems for Midterm 2 Test

(Th. April 20, 2017, 5-7pm or by arrangement)

April 16, 2017 *DRAFT*

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Be able to state and apply the following definitions:

1. Outer and inner Lebesgue measure, including all the definitions of the measures of special rectangles, special polygons, open sets, and compact sets.
2. Lebesgue measurable sets in terms of the outer and inner measures of the set. Be sure to include the case when the outer measure is infinite.
3. Describe a set that is not Legbesgue-measurable. (Proof not required.)
4. Fat Cantor sets
5. The Cantor-Lebesgue function.
6. An algebra and a σ-algebra.
7. The Borel σ-algebra. Know how to describe a set that is Lebesgue measurable, but not Borel measurable.
8. A null set.
9. A Lebesgue-measurable function – five equivalent definitions. Know proofs that the first 4 definitions are equivalent.
10. A simple function.
11. Class S of simple functions.
12. $\int s d\lambda$ for $s \in S$.
13. $\int f d\lambda$ for f measurable, nonnegative.
14. How to define an increasing sequence of simple functions which converge to an arbitrary measurable, nonnegative function. (The s_k's from the proof on p. 118.)
15. L^1, L^1 norm, convergence in L^1
16. f_-, f_+.
17. a.e.
18. Integrals over subsets of $\Re^n$.
19. Measure space (X, M, μ)
20. Step function on I
21. Upper Riemann integral, lower Riemann integral, and, when it exists, Riemann integral on I (for f bounded)
22. Lower and upper semicontinuous
23. $\underline{f}, \overline{f}$.
24. f_y, A_y

Be able to state the following results. Proofs of those with a 'p' should also be known. The lists of 'properties' below need not be memorized per se, but would be more likely to appear as true-false questions. Properties to be proved will be explicitly stated. Items 1-5 are for Test 1, not Test 2)

1. Properties $O1^p, O2^p, O3^p, O4^p, O5, O6$; $C1^p, C2^p, C3^p, C4$; $*1^p, *2^p, *3^p, *4^p, *5$
2. Theorem on Approximation (for sets in L_0 p. 45) and its Corollary (p. 46) (Test 1 only)
3. Theorem on Countable Additivity (for sets in L_0 p. 47)
4. Properties M1 – M4,M5^p, M6 – M10
5. $\lambda^*(TA) = |\det(T)|\lambda^*(A)$, $\lambda^*(z+A) = \lambda^*(A)$, $\lambda^*(tA) = t^n\lambda^*(A)$, and the analogues for inner measure and measure.
6. Proof that $c\int f(x)d\lambda = \int cf(x)d\lambda$ directly from the definition of Lebesgue integral, assuming you already know this result holds if f is a simple function.
7. Proof of property SC3: f is both LSC and USC at $x = a$ if and only if f is continuous at $x = a$
8. Proof of property SC4: f is LSC at x iff $\underline{f}(x) = f(x)$.
9. LICT
10. Fatou's Lemma^p (assuming LICT)
11. LDCT
12. The two approximation theorems of functions in L^1 by functions in C_c and C_c^∞.
13. The two Fubini theorems

Simple proofs that were part of previously done homework problems, or a step in such a proof, or a similar step or proof might also be asked. In particular,

- Ch 5: 2,6, 12c, 16abc (no proof required), 21
- Ch 6: 1,5,9
- Ch 7: 1,2,22

Examples:

1. A function that is Lebesgue-measurable, but not Lebegue integrable.
2. A function that is Lebesgue integrable, but not Riemann integrable
3. A sequence of integrable functions $\{f_k\}$ with the property that $\int \lim f_k d\lambda \neq \lim \int f_k d\lambda$.
4. An example of a sequence of functions $\{f_k\}$ and a function f for which f_k converges to f in L^1, but does not converge pointwise.
5. A set A measurable in $\Re^n$ but A_y is not measureable in $\Re^m$ $(n = l + m)$ for at least one y.