Real Analysis, Math 8201

List of Possible Problems for Midterm Test

(Thurs. March 2, 2017)

as of March 3, 2017

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Be able to state and apply the following definitions:

1. Construction of Lebesgue measure, including all the definitions of the measures of special rectangles, special polygons, open sets, compact sets, outer measure, inner measure, measureable sets with finite outer measure, measureable sets.

2. Lebesgue measurable sets in terms of the outer and inner measures of the set. Be sure to include the case when the outer measure is infinite.

3. A set that is not Legbesgue-measurable. Be able to write down such a set. (You are not required to prove that it is not measurable.)

Be able to state the following results. Proofs of those with a 'p' should also be known. The lists of 'properties' below need not be memorized per se, but would be more likely to appear as true-false questions. Properties to be proved will be explicitly stated.

1. Properties $O1^p, O2^p, O3^p, O4^p, O5, O6^p$

2. Properties $C1^p, C2^p, C3^p, C4$

3. Properties $*1^p, *2^p, *3^p, *4^p, *5$

4. Theorem on Approximation (for sets in L_0 p. 45) and its Corollary (p. 46)

5. Theorem on Countable Additivity (for sets in L_0 p. 47)

6. Properties M1 – M4,M5, M6 – M10. Proofs for M1 – M4.

7. $\lambda(z + tI) = t^n \lambda(I), \lambda^*(TA) = |detT|\lambda^*(A),\ \lambda^*(z + A) = \lambda^*(A),\ \lambda^*(tA) = t^n \lambda^*(A)$, ...

Simple proofs that were part of previously done homework problems, or a step in such a proof, or a similar step or proof might also be asked. In particular,

- Ch 1: 2g,10,18abc,19,43,44 (not originally assigned), also A bdd, f cts, f(A) not bdd. Compare with K compact, f cts, f(K) compact.

- Ch 2: 12,24 (or pf of Corollary), 26,29, 31

- Ch 3: 6 (forward direction only), 7, 8

- Ch 4: 1

- Sketch a subset A of $\Re^2$ illustrating $\lambda_*(A) < \lambda^*(A)$.

- Sketch disjoint subsets A and B of $\Re^2$ for which

  1. $\lambda^*(A \bigcup B) < \lambda^*(A) + \lambda^*(B)$. (cf Ch 4, 1)
  2. $\lambda_*(A \bigcup B) > \lambda_*(A) + \lambda_*(B)$. (cf Ch 4, 1)

- Sketch a figure illustrating M11: B is measurable, $A \subset B$, A is not measurable, but $\lambda^*(A) + \lambda_*(B \sim A) = \lambda(B)$.