Analysis of Ebola Disease Model with Hospitalization

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1 Abstract

Ebola is a highly lethal virus, which has caused 10702 total deaths in Africa since the 2013-14 winter through April 15, 2015 (World Health Organization). Using data from the epidemics (in Liberia in 2014), a differential equations model was built for the spread of Ebola, with many transmission rates and other epidemiological parameters. New in this study is the manipulation of the number of hospital beds, which increases the rate at which infected patients enter into the hospital. This is a key parameter which is regarded as a rapid institution of control [8]. Mathematica simulation tools were used to forecast the progression of the Ebola epidemic. For epidemic profiles identified in Liberia, increasing the number of beds increases the hospitalization rate, and reduces the number of individuals infected, as well as delaying the epidemic. In particular, it was found that increasing the number of the hospital beds in the range of (42200, 43000) leads to a mushrooming rise in survival.

2 Key Words

Disease model, Differential equation, Simulation, Dynamics, Hospitalization, Holling function, Epidemic, Ebola

3 Introduction

3.1 Background

Ebola is one of the most lethal viruses that infects humans. It seems to be able to avoid the human immune systems. Therefore, the Ebola virus can come on quickly and kill fast. People can be infected with Ebola by touching the blood or bodily fluids of an infected person or animal, or contacting an infected person's contaminated objects. The incubation period of the Ebola virus in the human is 2 to 21 days and the infectious period is 4 to 10 days [1]. Merler et al. pointed out that the routes of the transmission of the Ebola virus disease adopted from natural history model are: susceptible individuals acquire infection after contact with an infectious individual and become exposed without symptoms; at the end of the latent period infectious and symptomatic individuals can transmit infection to others [9]. Individuals might transmit infection during their funerals and are then removed from the model [3]. Additionally, diagnosis of Ebola is difficult. This can be why the virus breaks out widely without control. The current Ebola outbreak is the largest yet. Understanding the spread and control of this disease is clearly important. It has caused 10702 total deaths in Africa since the 2013-14 winter through April 15, 2015 (World Health Organization).

3.2 Approach

There are two standard ways to fight the Ebola disease; one is using good treatment to cure the patients, and another way is sending all patients to the hospital where they have a low contact rate. That is to say, in the hospital, the disease transmission rate is low compared with infectious individuals who are out of the hospital. Even today, there still are no efficient medical treatments to cure the disease. Therefore, controlling the spread the disease is the only thing we can do to fight the disease. As we want a low transmission rate of the spread of the disease, we should send more patients to the hospital. That is, we want to increase the hospitalization rate, where the hospitalization rate depends on the number of the hospital beds. A mathematical differential equation system model is used to simulate the trend of the disease to study how increasing the number of hospital beds could increase the hospitalization rate, and consequently increase the total survival number [11]. In the model, the total survival number contains three subparts; the first part is people who never catch the disease, the second part is people infected by the disease but who recover by their own immune system, and the third part is the patients who recover from the hospital.
The project consisted of the following parts:

1. Analyzing existing differential equations disease models which have been used to analyze the outbreak of Ebola and to get general sense of SIR disease modeling.

2. Adding more compartments to the basic SIR model to construct a SEIHFDR disease model which is more realistic but it still includes the unrealistic assumptions. Additionally, instead of setting all rates as constants, the hospitalization rate is a variable, changing with the number of hospital beds.

3. Creating a numerical differential equations simulation tool in Mathe-

matica by using the data from Liberia, which is one of the most severely affected areas, to simulate the trend of evolution of each group.

4. Understanding how varying the number of hospital beds in the model will change the outcome, modifying the behavior of the SEIHFDR model.

4 Assumptions

Dixon and Schafer suggested that the major challenges faced by all disaster areas in the efforts to control the outbreak include its wide geographic spread and weak health-care infrastructures [5]. However, in this model, we concentrated on the Liberia area and the wider geographic spread was ignored. The model included all the variables which seemed to be necessary to describe the outbreak of Ebola. I comprehensively considered all the situations, and the models were built on reality, using sensible, and useful criteria. However, to make the model to fit the general situation, I settled on several requisite assumptions.

1. According the periodic report from CDC, the outbreak happened in very short time. Thus, we ignored birth rate and background death and migration rates for population in our models. Therefore, the total population stays a constant.

2. Biologically, the Ebola virus is a kind of filoviridae family. They infect human beings by direct contact with infected patients or body fluids; even a dead body is still infectious. We assumed that all the infections, other than exposures, are infectious. Every individual is equally likely to be infected.

3. The population for Liberia is homogeneous.

4. The strain for Ebola does not vary before it is eradicated.

5. People in group R, group E and D are noninfectious. No quarantine will be provided to people in group E (Groups are defined in the Disease Model section below).

6. Patients who eventually arrive at group R and D do not infect other compartments. The people recovered from the Ebola never are reinfected by the disease.

7. Hospital capacity is limited, and it increases by adding more beds.

5 Notation

There was a total of thirteen transition rates enrolled in building our Ebola epidemic model. β_I indicated the transmission coefficient of infected people spreading the virus in the community, and β_H was the transmission coefficient at the hospital; β_F indicated the infection rate during funerals. The mean duration of the incubation period was $1/\ \alpha$. The mean duration from symptom onset to hospitalization was $1/\ \gamma_H$. Additionally, the mean duration of a patient being hospitalized to death was $1/\ \gamma_{DH}$, and $1/\ \gamma_D$ denoted the mean duration to death without hospitalization. A dead infected body should wait the mean duration of $1/\ \gamma_F$ to burial.

Description	Parameter
Contact Rate, Community	β_I
Contact Rate, Hospital	β_H
Contact Rate, Funeral	β_F
Incubation Period	$1/\ \alpha$
Time until Hospitalization	$1/\ \gamma_H$
Time from Hospitalization to Death	$1/\ \gamma_{DH}$
Duration of Traditional Funeral	$1/\ \gamma_F$
Duration of Infection	$1/\ \gamma_I$
Time from Infection to Death	$1/\ \gamma_D$
Time from Hospitalization to Recovery	$1/\ \gamma_{IH}$
Daily probability a Case is Hospitalized	τ
Case Fatality Rate, Unhospitalized	δ_1
Case Fatality Rate, Hospitalized	δ_2

Table 1: Model Parameters for the Ebola Epidemic in Liberia, 2014 [10].

Some infectious patients recovered without hospitalization, and $1/\ \gamma_I$ rep-

resented the mean duration of these infectious period for survivors; While, for those patients who were hospitalized, $1/\ \gamma_{IH}$ denoted the mean duration from hospitalization to end of infectiousness for cure; The probability of a case being hospitalized per day was τ; δ_1 and δ_2 represented the infected case fatality rate with and without hospitalization, respectively.

6 Disease Model

In order to optimize the eradication of Ebola, we have to know about how fast this disease spreads, how many people will die, and how long this epidemic will take to destroy Liberia; in other words, we should understand this mystery killer mathematically.

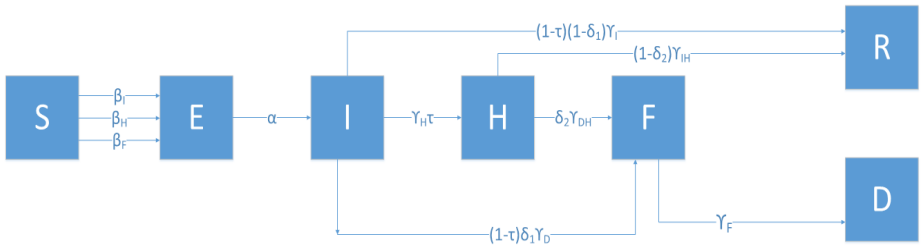

Figure 1: Compartmental flow of a Ebola mathematical model

Mathematical modeling has emerged as an important tool for gaining understanding of the dynamics of the spread of infectious diseases [2]. We model the progress of an epidemic in a large population comprising many different individuals by keeping track of the number of individuals within the subgroups, which are called compartments [4]. Differential equations are written to model the rates at which people go from one compartment to another [7]. An improved model (SEIHFDR), is shown in the Figure 1. Susceptible individuals (S) will become exposed (E) after contact with the infectious patients, and then turn to infectious individuals (I) after an incubation period of the Ebola disease. Additional sub-groups: hospitaliza-

tion (H) represents the number of hospitalized cases, and funeral (F) is the number of cases who are dead but still can infect other people who come in contact with their bodies prior to burial. After funerals, all the cases will be transmitted to the death group (D) and removed from the chain of transmission. Patients can recover (R) from Ebola either on their own or under hospital control. In the model, natural death, birth, and migration are ignored.

$$\frac{dS}{dt} = -\frac{\beta_I SI + \beta_H SH + \beta_F SF}{N} \tag{1}$$

$$\frac{dE}{dt} = \frac{\beta_I SI + \beta_H SH + \beta_F SF}{N} - \alpha E \tag{2}$$

$$\frac{dI}{dt} = \alpha E - \left[\gamma_H \tau + \gamma_I \left(1-\tau\right)\left(1-\delta_1\right) + \gamma_D \left(1-\tau\right)\delta_1\right] I \tag{3}$$

$$\frac{dH}{dt} = \gamma_H \tau I - \left[\gamma_{DH}\delta_2 + \gamma_{IH}\left(1-\delta_2\right)\right] H \tag{4}$$

$$\frac{dF}{dt} = \gamma_D \left(1-\tau\right)\delta_1 I + \gamma_{DH}\delta_2 H - \gamma_F F \tag{5}$$

$$\frac{dD}{dt} = \gamma_F F \tag{6}$$

$$\frac{dR}{dt} = \gamma_I \left(1-\tau\right)\left(1-\delta_1\right) I + \gamma_{IH}\left(1-\delta_2\right) H \tag{7}$$

7 Simulation and Analysis

Data for Ebola cases was collected from public data released by the World Health Organization, as well as the Ministries of Health of the afflicted countries. For this paper, the parameters are fixed from the 2014 Ebola epidemic in Liberia shown the table 2 below.

The total population in Liberia in 2014 was around 4,302,475, with 8,475 infectious patients; the rest are the initial susceptible population. The number of hospital beds is manipulated. According to the graph the susceptible

Description	Parameter	Fitted Values
Contact Rate, Community	β_I	0.160
Contact Rate, Hospital	β_H	0.062
Contact Rate, Funeral	β_F	0.489
Incubation Period	$1/\ \alpha$	12 days
Time until Hospitalization	$t_h = 1/\gamma_H$	3.24 days
Time from Hospitalization to Death	$1/\ \gamma_{DH}$	10.07 days
Duration of Traditional Funeral	$1/\ \gamma_F$	2.01 days
Duration of Infection	$1/\ \gamma_I$	15.00 days
Time from Infection to Death	$1/\ \gamma_D$	13.31 days
Time from Hospitalization to Recovery	$1/\ \gamma_{IH}$	15.88 days
Daily probability a Case is Hospitalized	τ	0.197
Case Fatality Rate, Unhospitalized	δ_1	0.500
Case Fatality Rate, Hospitalized	δ_2	0.500

Table 2: Model parameters and fitted values for a model of an Ebola epidemic in Liberia, 2014 [10].

group immediately decreases and at the same time the infected groups numbers begin to increase. The number of the death and recovery begin to rise and continues to increase until a certain percentage of the population. The death groups peak is around 1.7×10^6, and recovery groups peak is around 2.1×10^6, respectively. Around the same time, susceptible groups curve is also hitting a plateau, which is just about 3×10^5 people left. The exposed group and the infected group both head to zero.

8 Analysis of Hospitalization: An Improved Model

Instead of the hospitalization rate γ_H being constant, we assume a Holling type II functional response. This function is given below, where b denotes

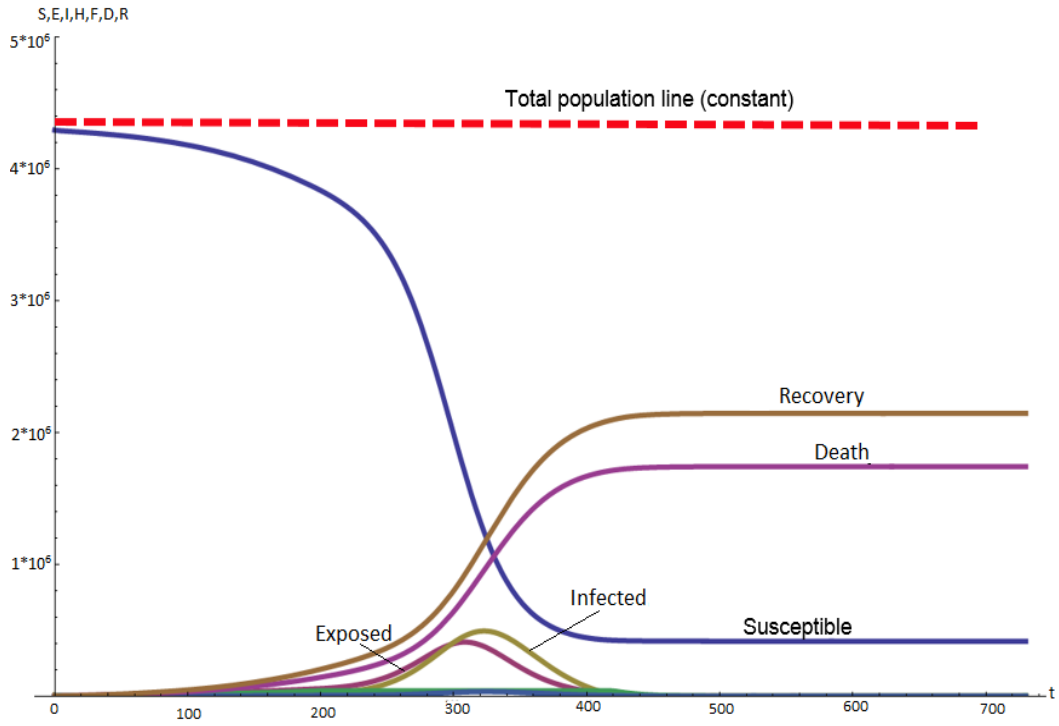

Figure 2: Simulation of the compartmental models with Mathematica. Each compartment is graphed as a function of t. Note that parameters were token from table 2 and the number of beds is fixed at 40000.

number of the hospital beds. The slope of γ_H when b is close to H is ϵ. We note that the number of hospitalized patients (H) is always less than the number of available beds (b). The hospitalization rate for large b is $1/t_h$. See the graph of γ_H in left side of figure 3.

$$\gamma_H = \frac{\epsilon\ (b - H)}{1 + \epsilon\ t_h\ (b - H)} \tag{8}$$

In the improved model, the rate at which patients are hospitalized will depend on the number of open beds in the hospitals. By the form of equation (8), as hospital capacity approaches infinity (increasing b by adding more beds to the hospital), the rate of hospitalization γ_H approaches the constant $1/\ t_h$. Consequently, the total number of people surviving will not increase by increasing the number of hospital beds after the hospitalization rate approaches to a certain value, shown in the right side of Figure 3 below.

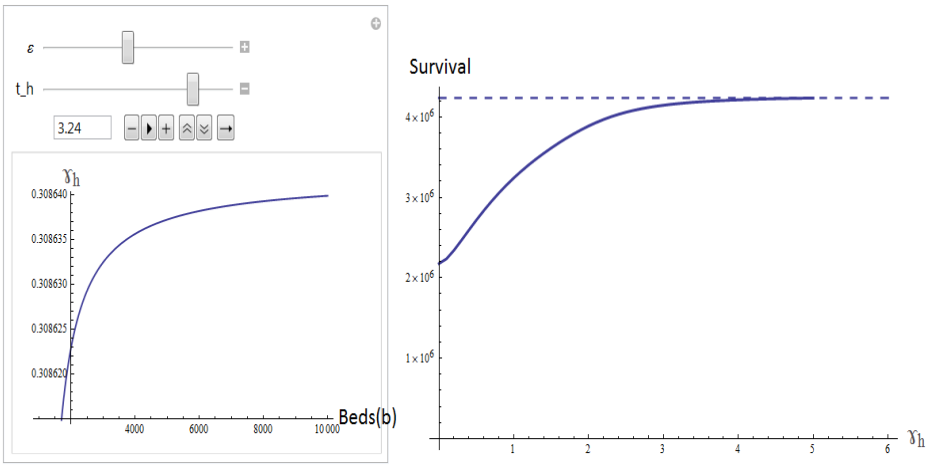

Figure 3: Left: Hospitalization rate, γ_H , as a function of hospital beds, b. Right: Total survival as a function of γ_H. Enlarging the capacity of the hospital seems ineffectual after some number of beds(b)

By manipulating the number of the beds and fixing the other parameters, as we increase the number of beds, the epidemic will delay. Additionally,

people who are infected and die due to Ebola will both decrease.

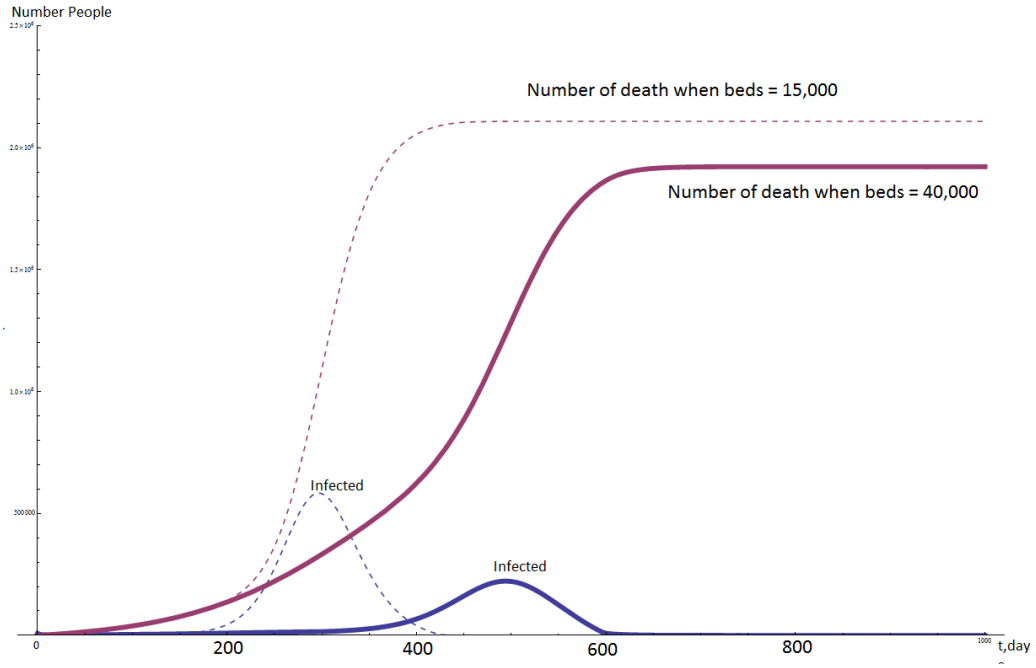

Figure 4: The epidemic is delayed by increasing beds(b). The dashed lines represent the disease when number of beds equal 15,000 and solid lines represent the number of beds equal 40,000.

When adding more beds, more patients will be sent to the hospital, where the contact rate to infect susceptible individuals is lower. In the bottom of Figure 6, it shows that more hospital beds lead the increase of the total survival population. Note that, the total population surviving contains people never infected by the virus, shown at the bottom of Figure 5, people recovered by themselves, shown at the top of Figure 6, and people cured in the hospital, shown at the top of Figure 5.

In particular, increasing the number of the hospital beds in the range of (42200, 43000) leads to a mushrooming rise of the total surviving shown in

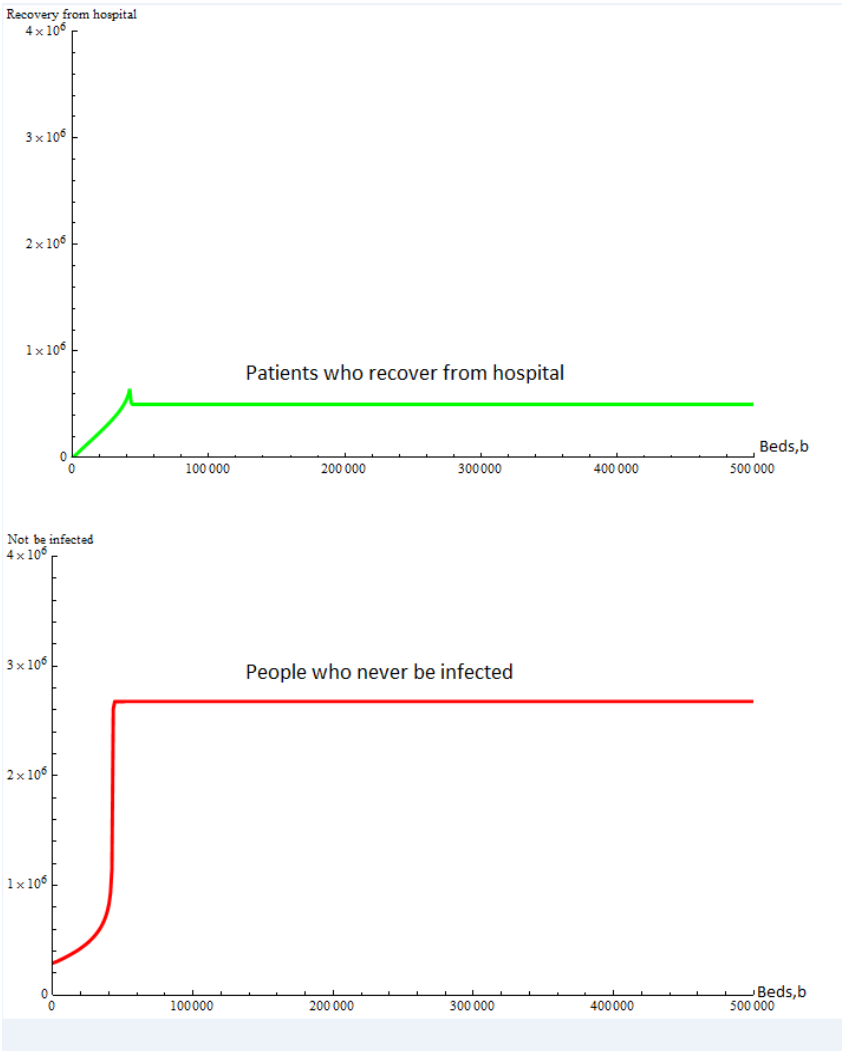

Figure 5: Amount of people who survive as number of beds(b) changes

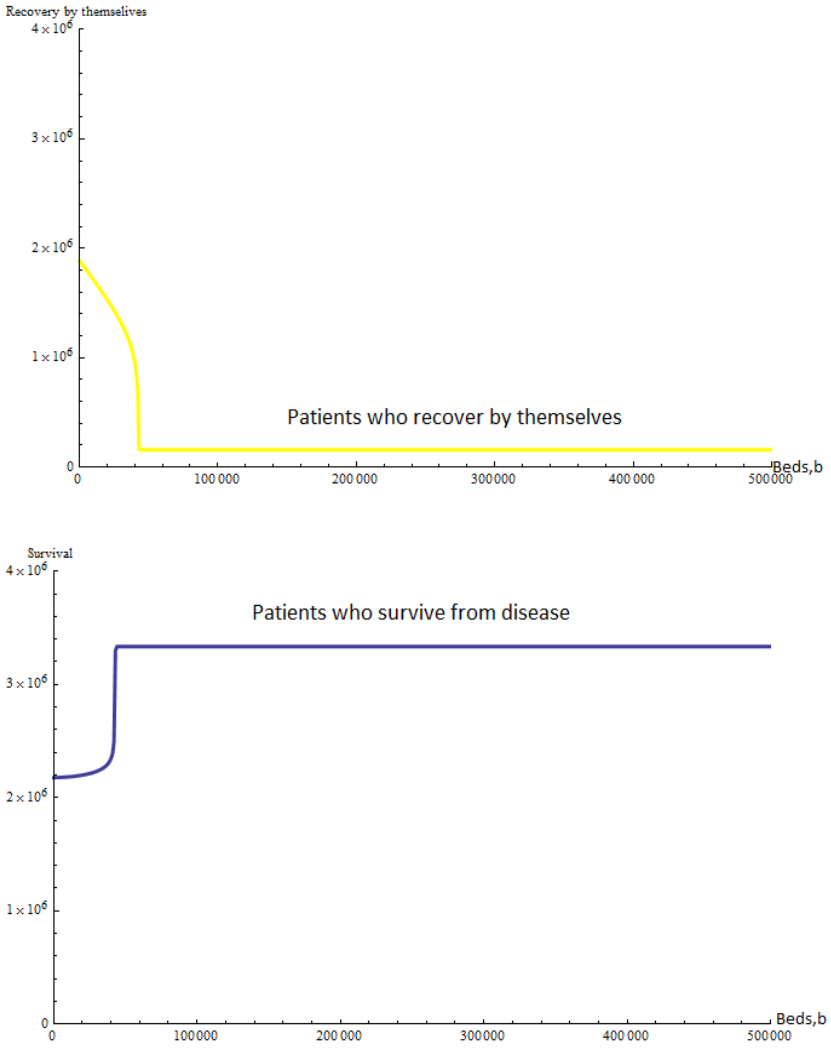

Figure 6: Amount of people who survive as number of beds(b) changes

figure 7. Note that, the total who recover from the disease is decrease, due to fewer being infected.

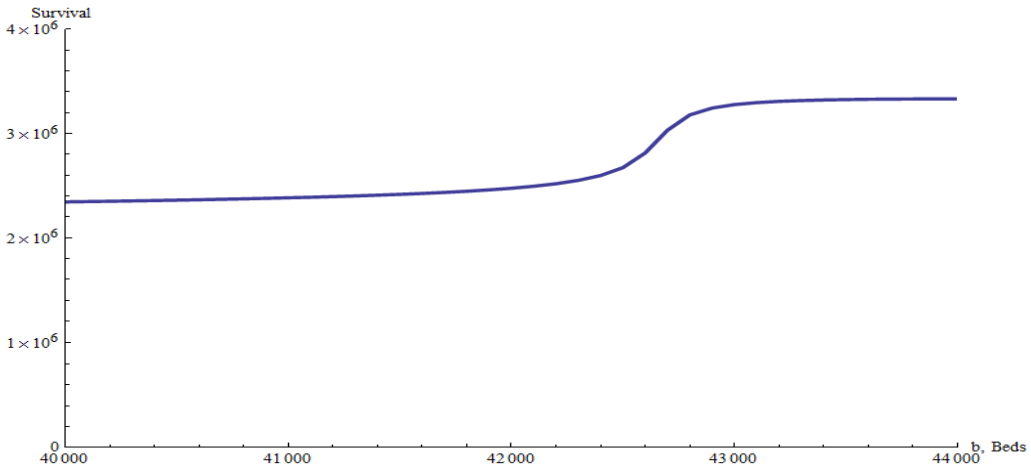

Figure 7: Zoom of the part of Survival vs Beds graph in bottom of Figure 6, where a small number of increase beds(b) effects a significant change in the total amount who survive.

9 Conclusion

Mathematical models are widely used to examine, explain and predict the dynamics of infectious disease transmission, and models of specific diseases of global import have played a vital role in developing public health strategies for control and prevention [6]. The Ebola disease model relies on vital parameters and they all play a part in determining epidemic evolution. Based on the simulation using known data from Liberia, as we increase the capacity of the hospital, the hospitalization rate is increased. However, after exceeding 43,000 beds in the hospital, the hospitalization rate approaches a constant. As the hospitalization rate increases, the total survival amount increases. However, as the hospitalization rate approaches its maximum, the

survival stays relatively constant. Therefore, it is a poor use of resources to keep increasing the number of the beds. From the result of the simulation, it can be concluded the number of hospital beds in Liberia in the range of (42200, 43000) will be the most efficient use of resources to control the outbreak of the Ebola disease. In that range, a small increase in the number of beds result in a significant increase in total number of people who survive.

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