Math 3298 – Spring 2013
Exam 3 - 75 points
Name____________________

Show your work for full credit!

Some Multiple/Line Integral Stuff:

\[ A(S) = \iint_D \sqrt{1 + \left( \frac{\partial z}{\partial x} \right)^2 + \left( \frac{\partial z}{\partial y} \right)^2} \, dA \]

\[ \iiint_E f(x, y, z) \, dV = \int_c^d \int_a^b \int_f^g f(\rho \sin \phi \cos \theta, \rho \sin \phi \sin \theta, \rho \cos \phi) \rho^2 \sin \phi \, d\rho \, d\theta \, d\phi \]

\[ \iint_R f(x, y) \, dA = \iint_S f(x(u, v), y(u, v)) \left| \frac{\partial (x, y)}{\partial (u, v)} \right| \, du \, dv \]

\[ \left| \frac{\partial(x, y)}{\partial(u, v)} \right| = \left| \begin{array}{cc} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{array} \right| \]

Parametric line segment: \( \mathbf{r}(t) = (1-t)\mathbf{r}_0 + t\mathbf{r}_1 \quad 0 \leq t \leq 1 \)

1. Determine the surface area of the portion of the paraboloid \( z = 9 - x^2 - y^2 \) above the \( xy \)-plane.

2. Rewrite the integral \( \iiint_{E} f(x, y, z) \, dz \, dy \, dx \) as iterated integrals in the order \( dz \, dx \, dy \) and \( dx \, dy \, dz \).

3. Evaluate \( \iiint_E z \, dV \) where \( E \) is the solid region between the spheres \( x^2 + y^2 + z^2 = 1 \) and \( x^2 + y^2 + z^2 = 9 \) in the first octant.
4. Use the transformation, \( u = x + y \) and \( v = x - y \) to change the variables for the integral
\[
\iint_R \ln(x + y) \, dA
\]
where \( R \) is defined as the interior of the square with vertices \((2,1)\), \((3,2)\), \((2,3)\) and \((1,2)\). Write an equivalent integral in terms of \( u \) and \( v \) over the transformed region. **Do not evaluate the integral.**

5. Evaluate the line integral \( \int_C xe^{yz} \, ds \) where \( C \) is the line segment joining the points \((0,0,0)\) to \((2,2,2)\).