

H. S. M. Coxeter and Tony Bomford's Colored Hyperbolic Rugs

Douglas Dunham

Department of Computer Science

University of Minnesota, Duluth

Duluth, MN 55812-2496, USA

E-mail: ddunham@d.umn.edu

Web Site: <http://www.d.umn.edu/~ddunham/>

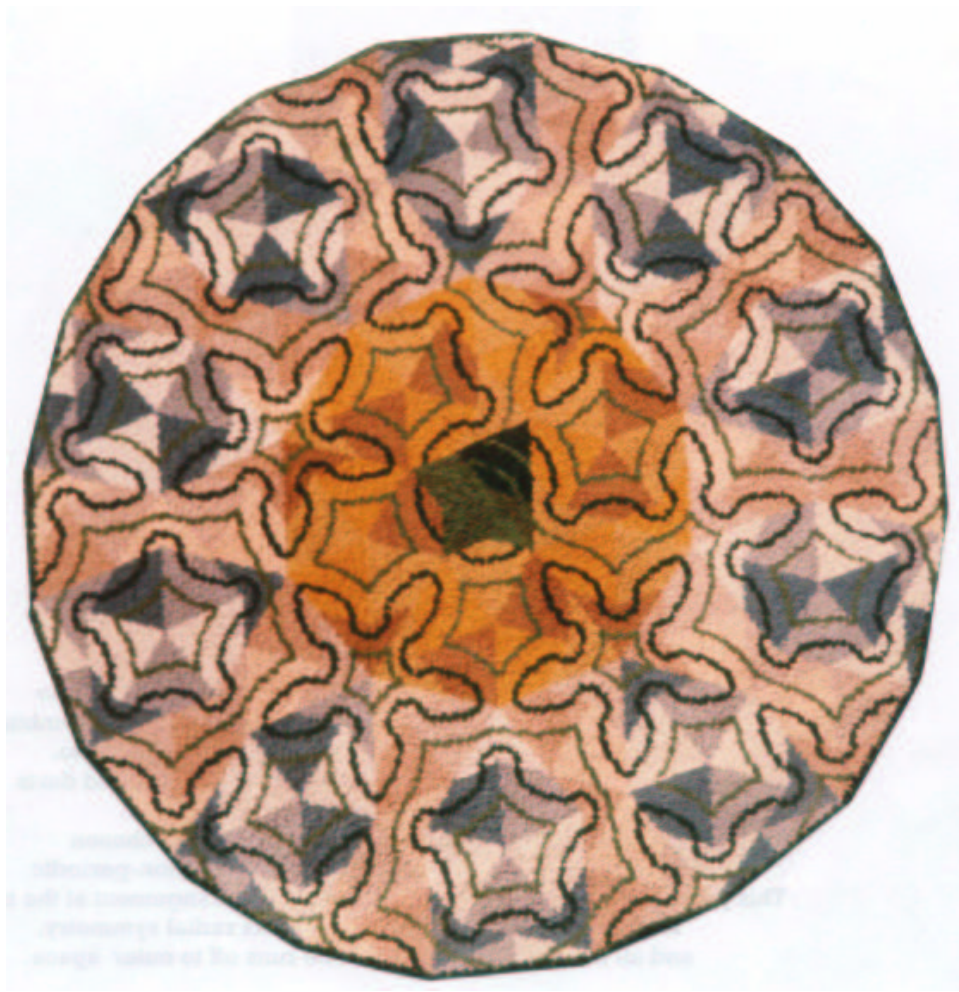




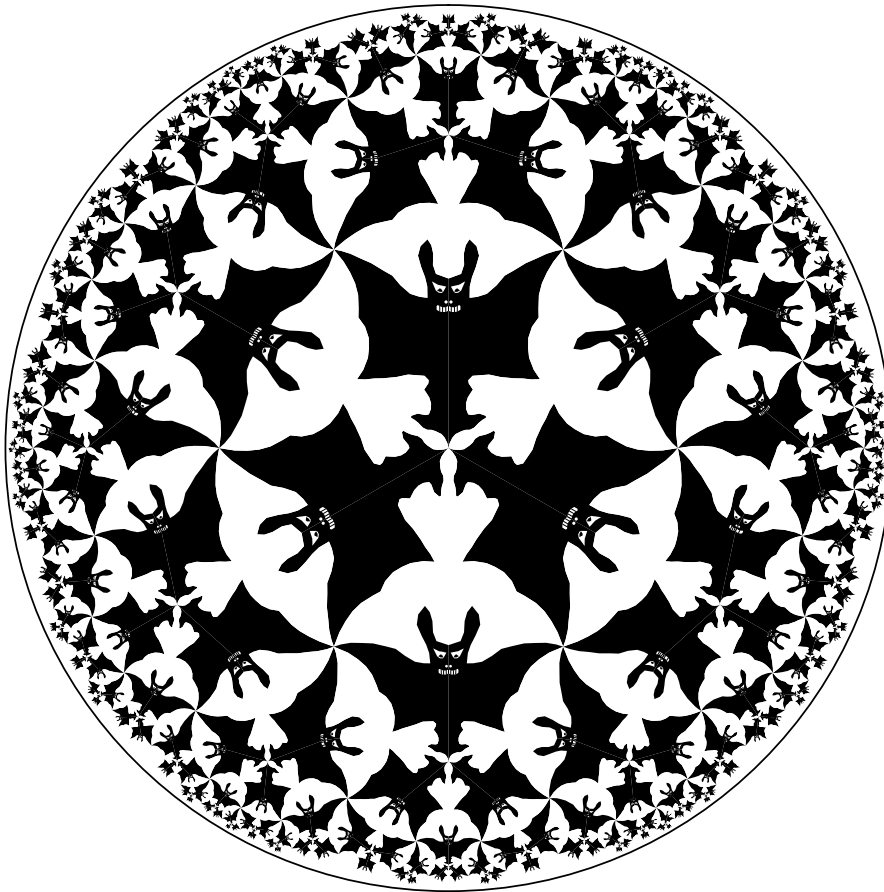
Tony Bomford (1/17/1927 – 5/10/2003), geodesist and polymath.



Rug #1, a commercial design: “Kabistan 2295” by Paton & Baldwin.



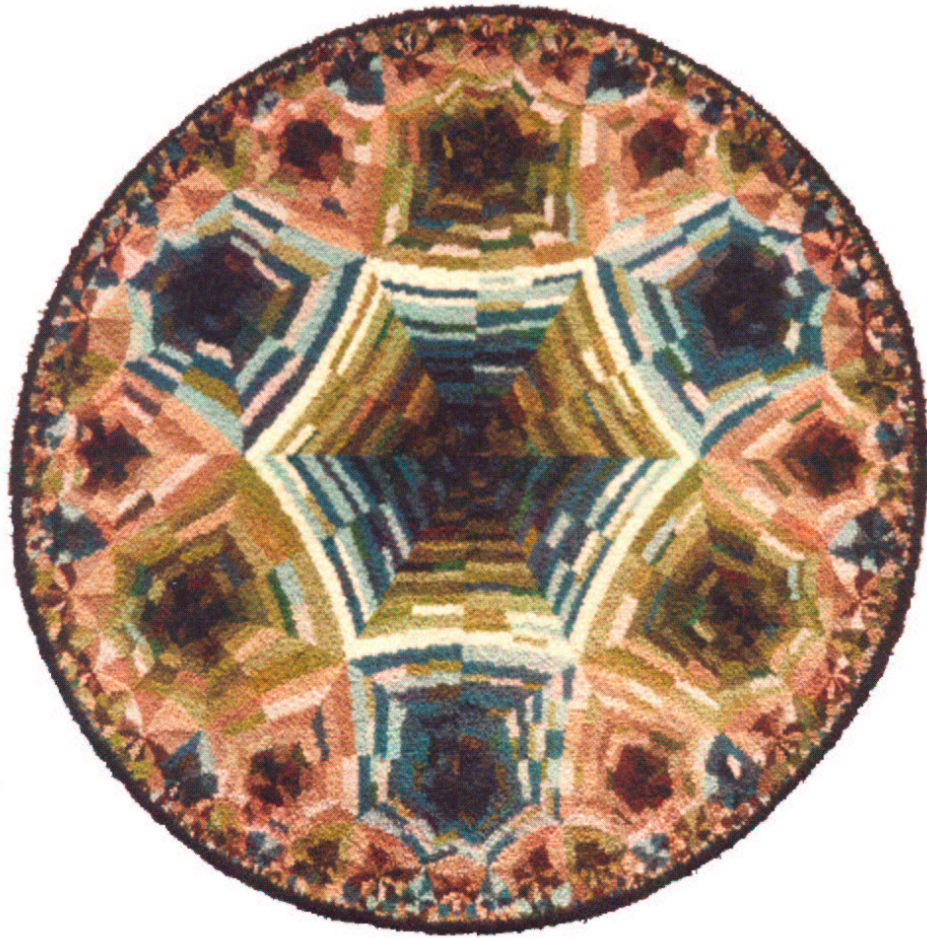
Rug #10, a tiling based on Penrose's "Kites and Darts".



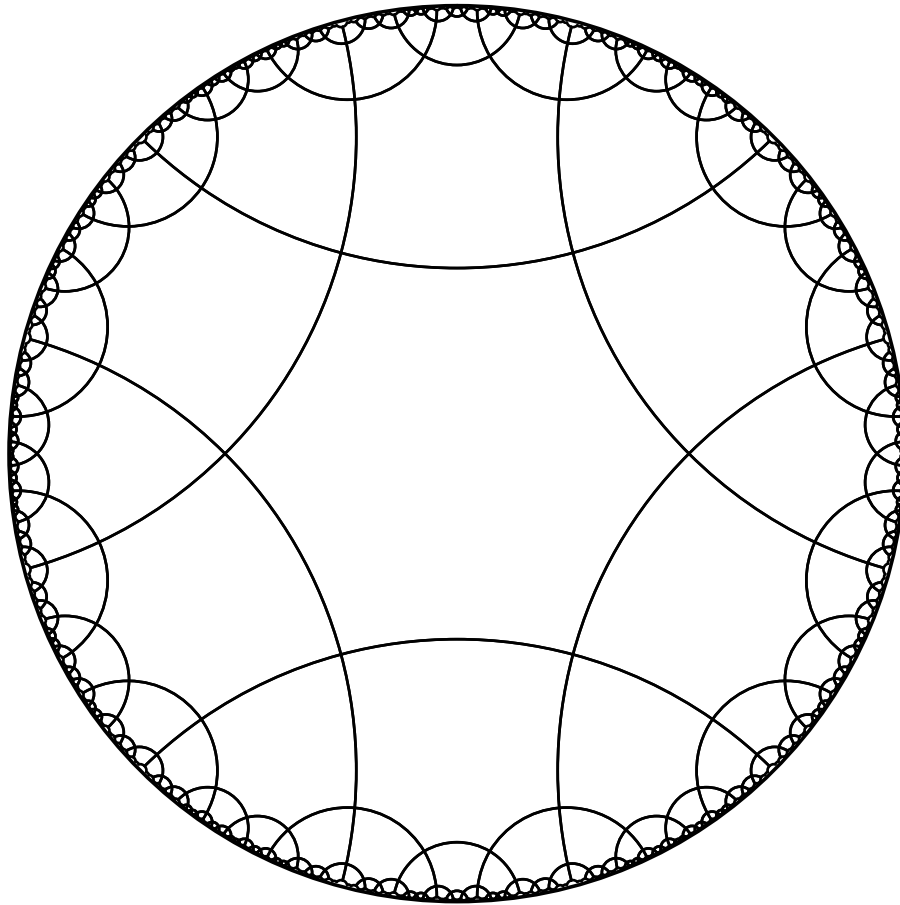
**Bomford's first inspiration from an article by
Coxeter: M.C. Escher's hyperbolic pattern *Circle
Limit IV* in the chapter *Angels and Devils* of *The
Mathematical Gardner*.**



Rug #12: *Hyperbolic Spiderweb.*



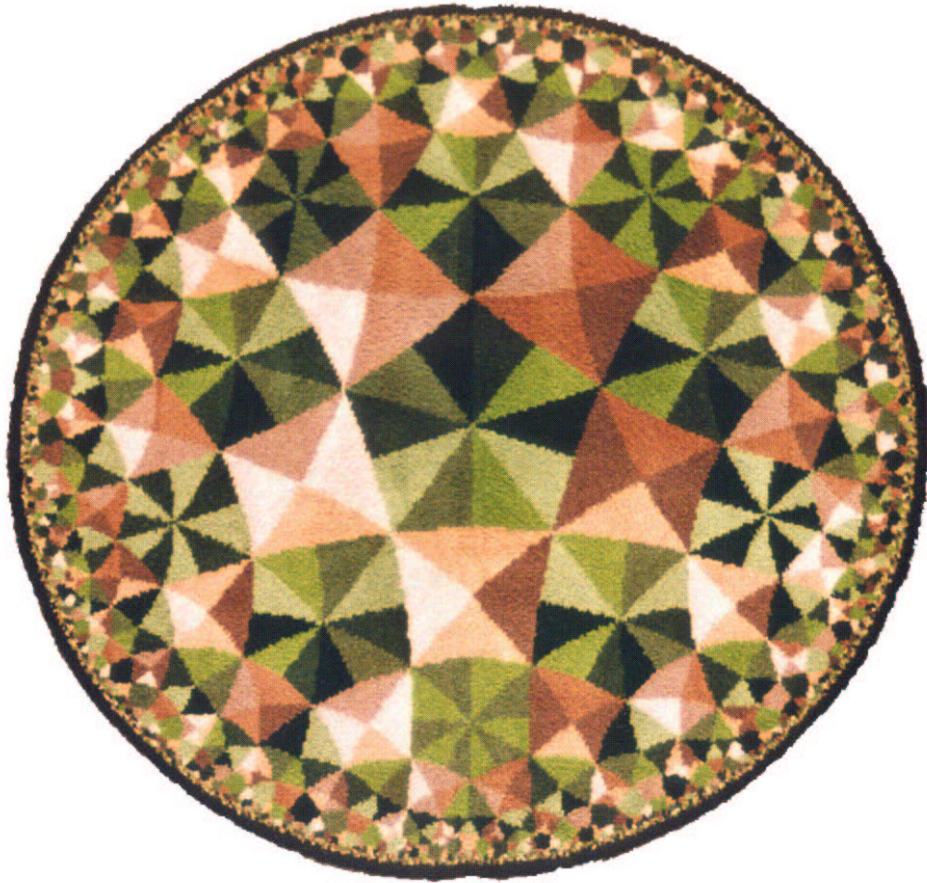
Rug #13: *Hyperbolic Lagoon.*



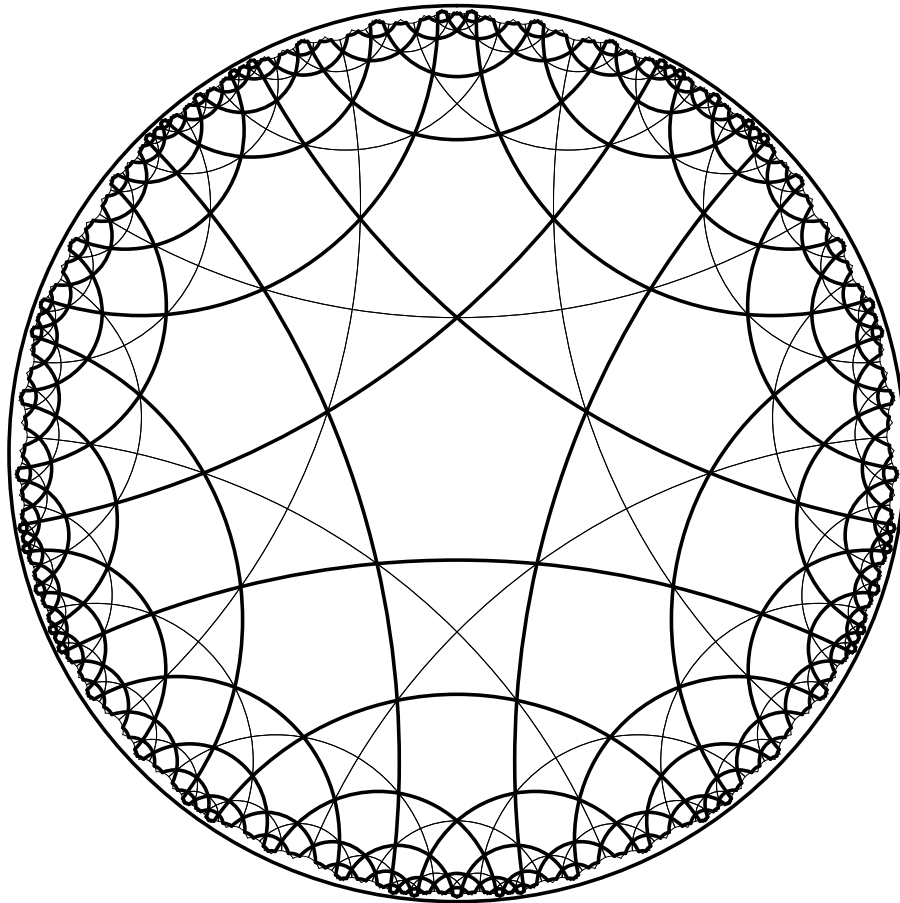
The tessellation $\{6,4\}$.

In general $\{p, q\}$ denotes the *regular tessellation* by regular p -sided polygons meeting q at a vertex.

The tessellation is hyperbolic if $(p - 2)(q - 2) > 4$.



Rug #15: *Squares and Pentagons.*



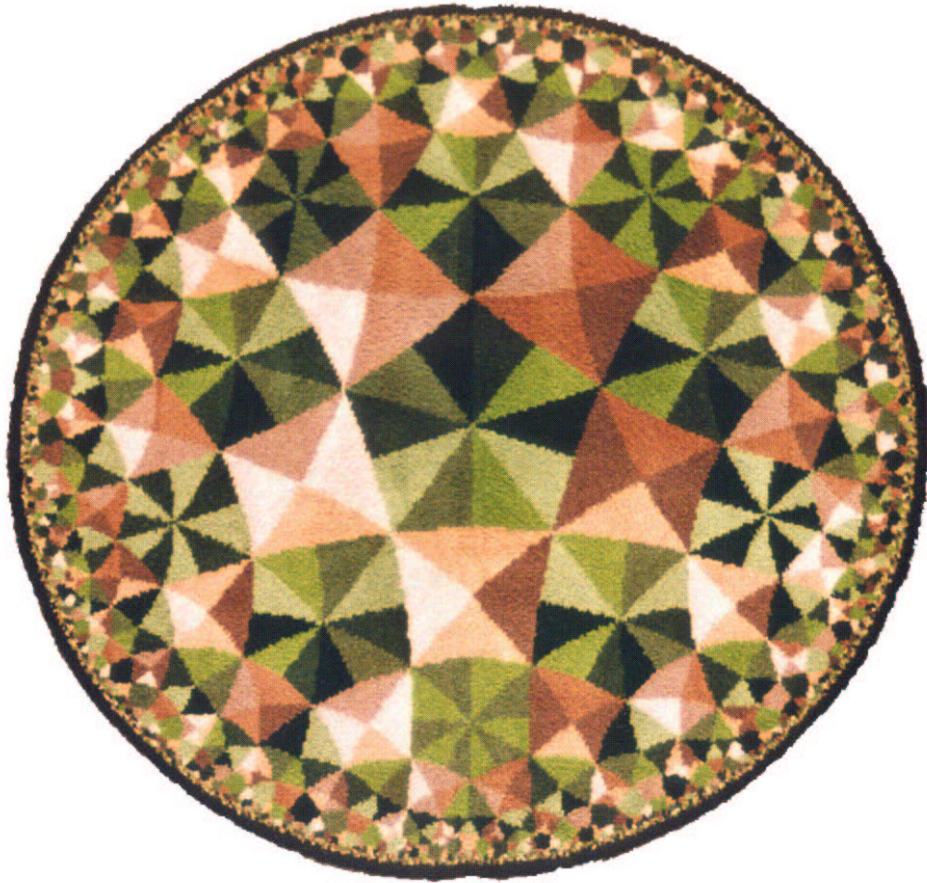
The tiling (5.4.5.4) (bold lines) superimposed on the tessellations {5,4}.

In general one can obtain the semi-regular (or uniform) tiling **(p.q.p.q)** from the tessellation $\{p,q\}$ by connecting midpoints of adjacent sides of the p-sided polygons.

A *color symmetry* of a pattern is a symmetry of the uncolored pattern that permutes the colors — taking all parts of one color to parts of another (possibly the same) single color.

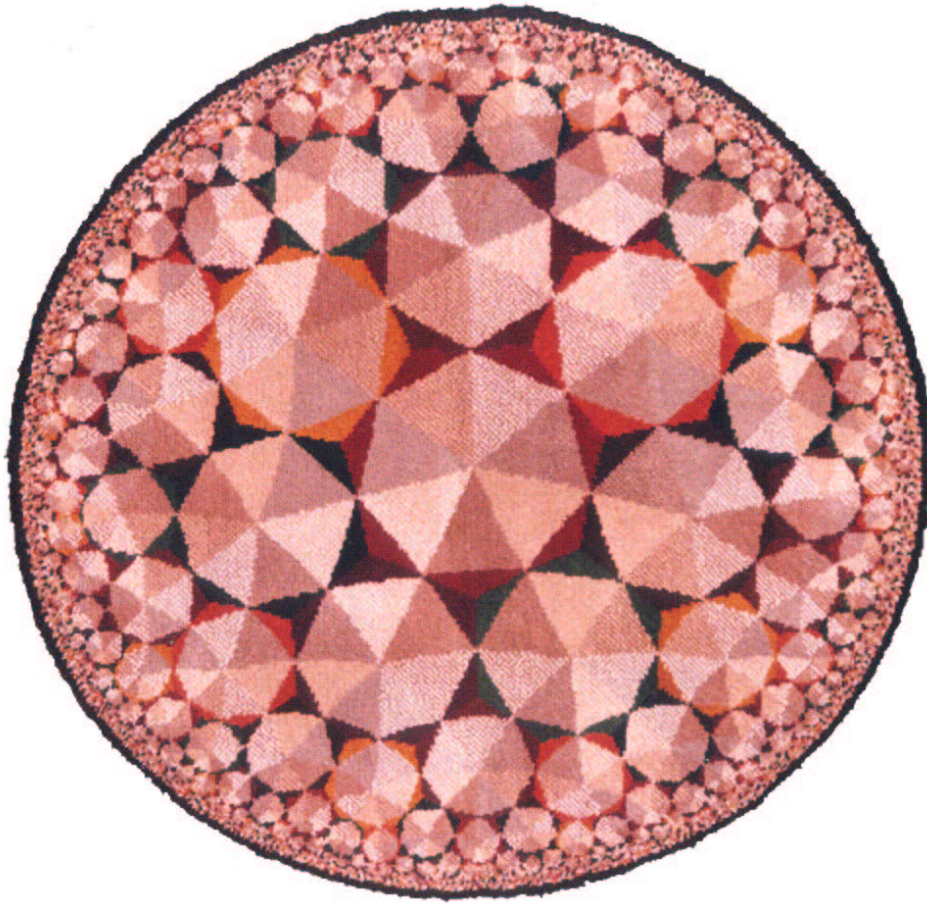


Rug #12 (*Hyperbolic Spiderweb*) exhibits color symmetries.

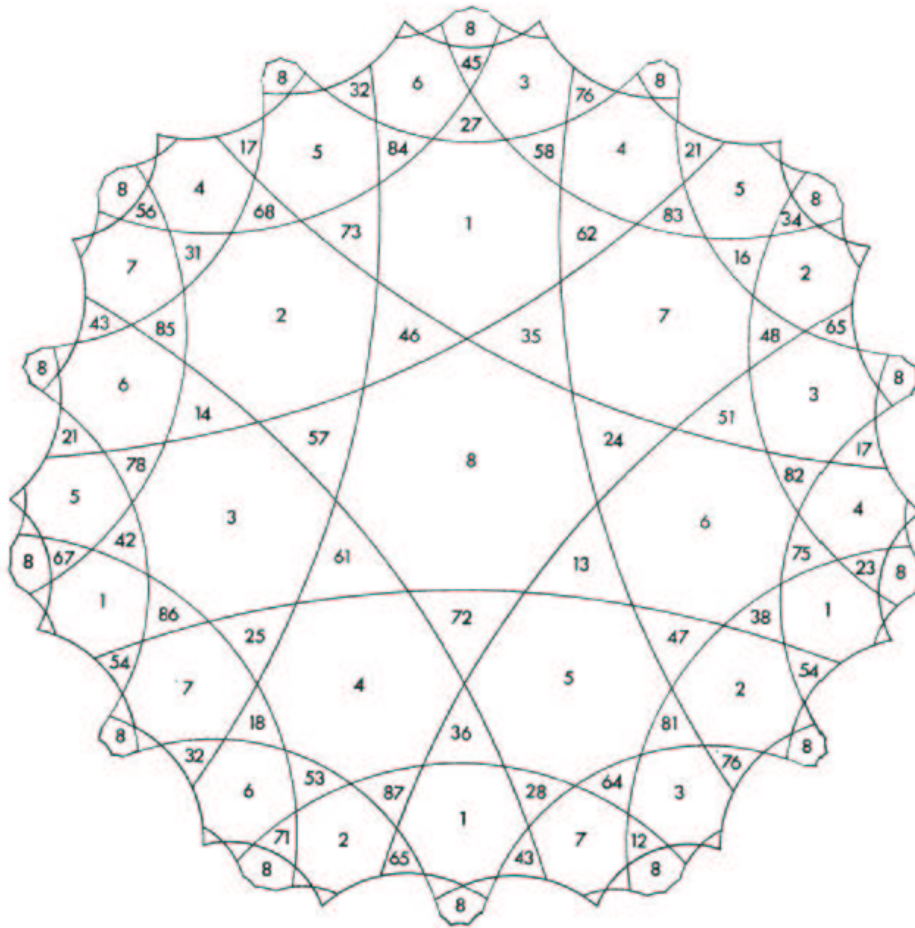


Rug #15 (*Squares and Pentagons*) also exhibits color symmetries.

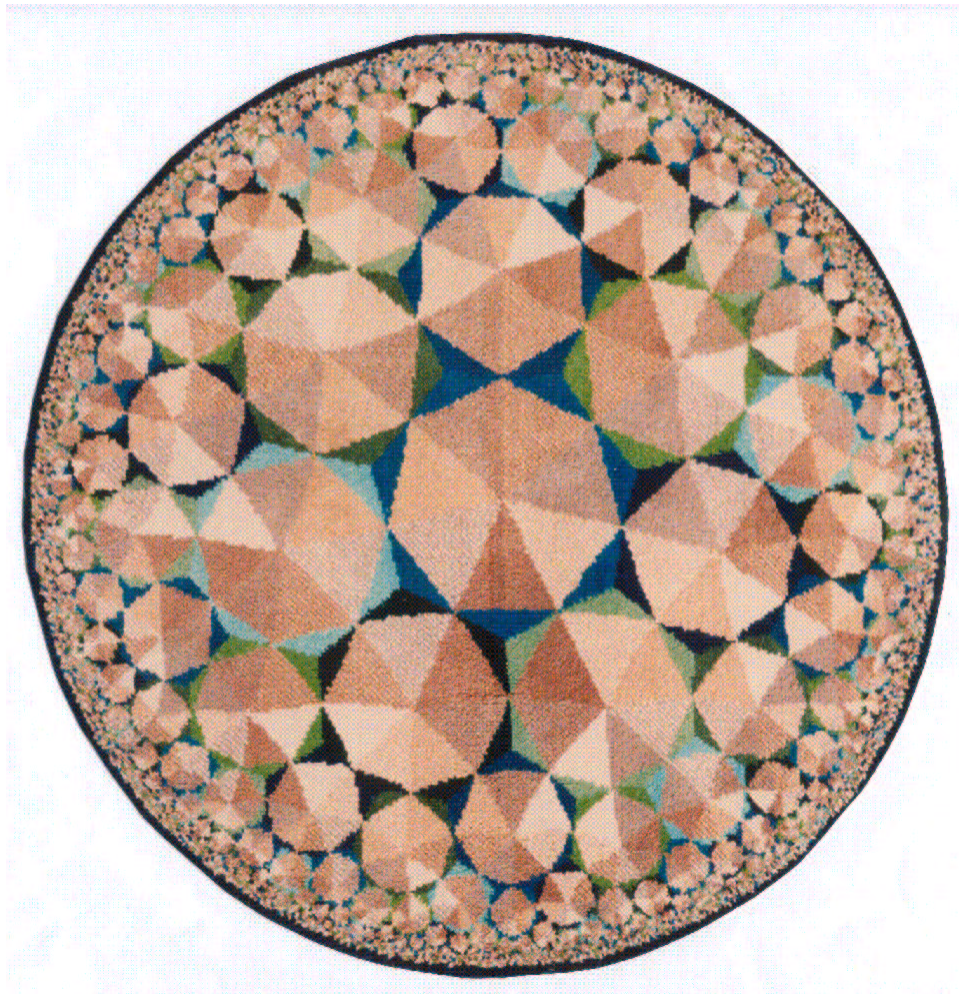
A pattern has *perfect color symmetry* if every symmetry of the pattern is a color symmetry.



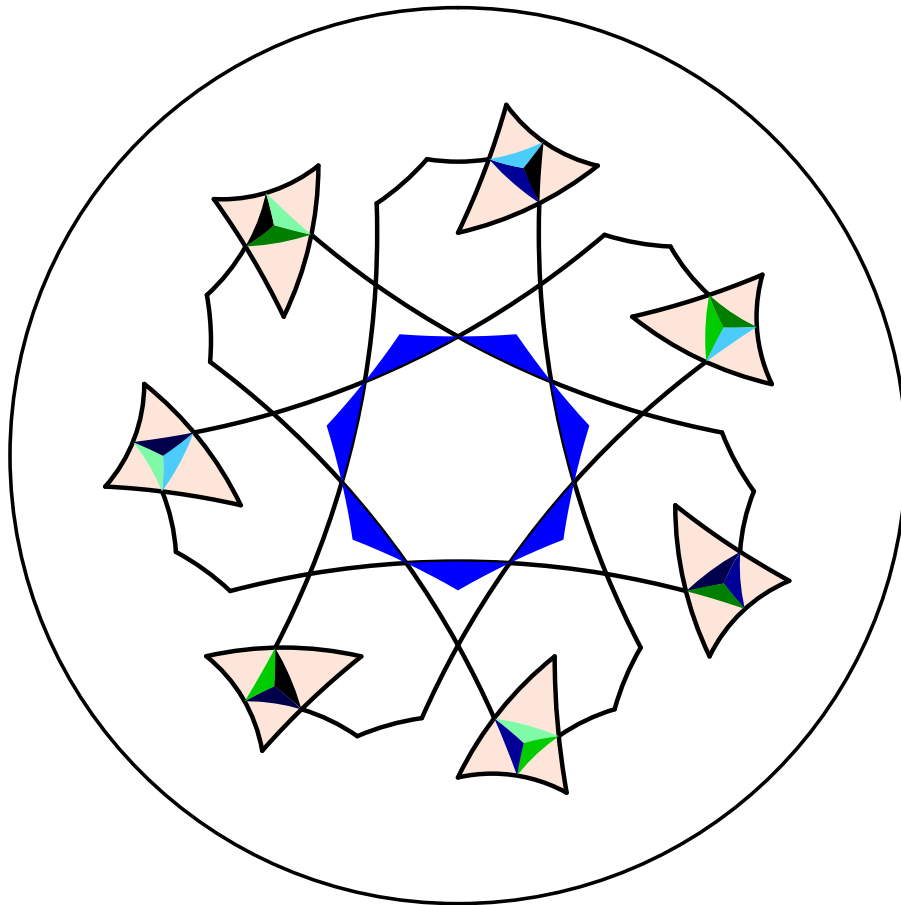
Rug #16, *Triangles and Heptagons*, exhibits perfect color symmetry if we consider each “bright color” and its corresponding tan to be a single color.



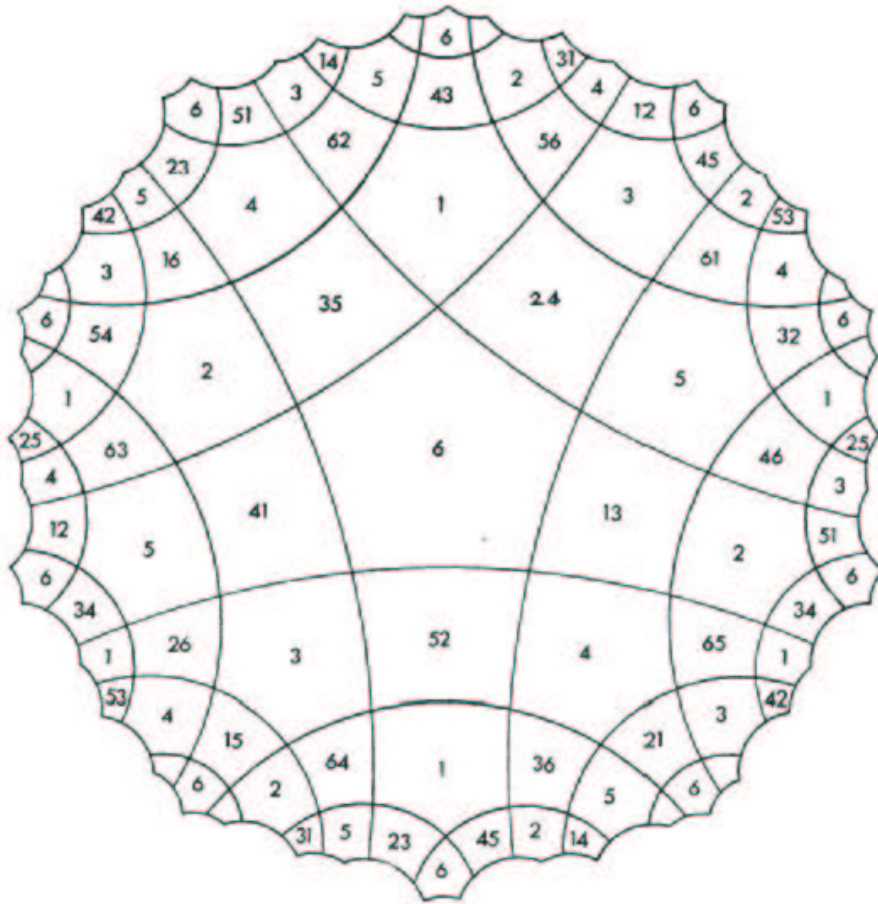
Bomford's second inspiration from an article by Coxeter: Figure 10, an annotated (7.3.7.3) tiling in *Regular compound tessellations of the hyperbolic plane* in the Proceedings of the Royal Society of London, Series A 278 (1964).



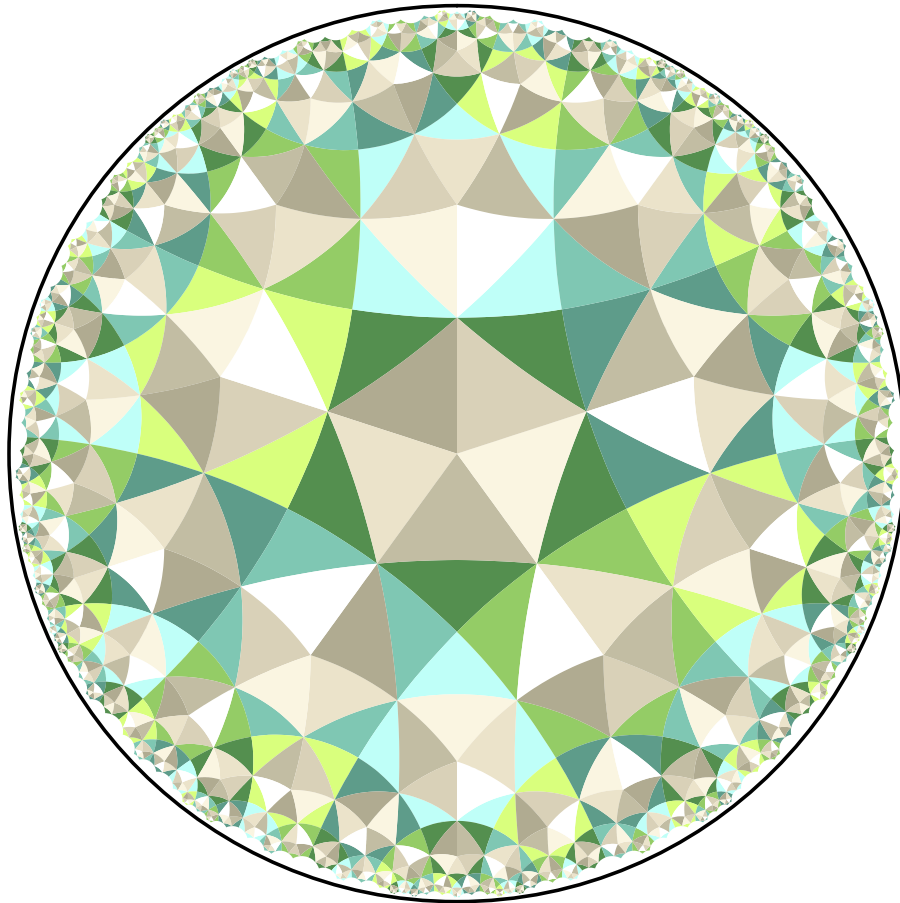
Rug #17 exhibits the same kind of color symmetry as Rug #16 and is also based on the (7.3.7.3) tiling.



Some corresponding triangles in Rug #17 whose tan corresponds to the blue around the central heptagon.



**A “missed” inspiration from an article by Coxeter:
 Figure 8, an annotated (5.4.5.4) tiling in *Regular
 compound tessellations of the hyperbolic plane* in the
 Proceedings of the Royal Society of London, Series A
 278 (1964).**



A rug design based on Coxeter's Figure 8 and having the same kind of color symmetry as Bomford's Rugs 16 and 17.

Future Work

Make the hooked rug shown in the last slide!