Hyperbolic Vasarely Patterns

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Abstract

Victor Vasarely was one of the founders of the Op Art movement in the 1930's. Among other art works, he created a number of patterns based on a grid of squares, sometimes containing circles. These images are basically Euclidean. In this paper I show some hyperbolic patterns that were inspired by Vasarely's grid patterns.

1. Introduction

The Hungarian artist Victor Vasarely designed many patterns based on the square grid. Figure 1 is a hyperbolic pattern of "squares" inspired by Vasarely's square grid patterns. Ever since I became involved in

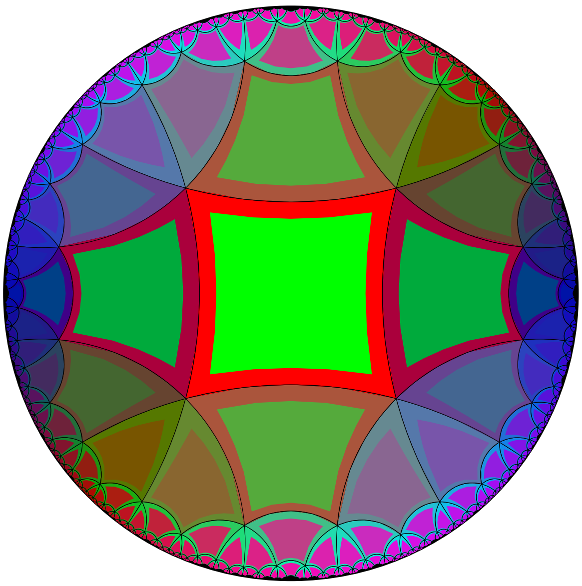

Figure 1: A hyperbolic pattern of "squares" based on the {4,6} grid.

hyperbolic geometry, I have thought about creating hyperbolic Vasarely patterns. So it seemed particularly appropriate to do it now, since the Bridges Conference will be held in Pécs, Hungary, Vasarely's birthplace.

We begin with a very short biography of Vasarely. Then to understand the hyperbolic patterns, we review a bit of hyperbolic geometry and regular tessellations. Next we discuss hyperbolic patterns based on "square" grids, containing either framed squares or circles. Then we show some random patterns. We also consider "hexagon" patterns, though this endeavor was not quite as successful. Finally, we indicate possible directions of further research.

2. A Brief Biography of Vasarely

Victor Vasarely was born Vásárhelyi Győző in Pécs April 9, 1906. In 1927 he abandoned medical studies and took up painting. He moved to Paris in 1930 and spent much of his life there. In 1937 he created *Zebras*, generally considered to be one of the first pieces of Op Art. Starting in the late 1940's his art became more and more geometric. From 1965 to 1979 he published many of his works in a four book sequence: *Vasarely I*, *Vasarely II*, *Vasarely III*, and *Vasarely IV* [3]. After a very productive and influential career, he died in Paris March 15, 1997 at age 90. More of his works may be viewed on his official web site [2].

3. Hyperbolic Geometry and Regular Tessellations

The patterns that I have created can be interpreted as patterns in the hyperbolic plane, and specifically in the *Poincaré disk* model of hyperbolic geometry. The hyperbolic points in this model are represented by Euclidean points within a bounding circle. Hyperbolic lines are represented by (Euclidean) circular arcs orthogonal to the bounding circle (including diameters). The hyperbolic measure of an angle is the same as its Euclidean measure in the disk model (we say such a model is *conformal*), but equal hyperbolic distances correspond to ever-smaller Euclidean distances as figures approach the edge of the disk. In the rest of the paper, we will use the word *square* to mean any equilateral equiangular quadrilateral. Figure 1 is composed of such squares, each one having an interior color and a contrasting "frame"; the edges of the squares lie along hyperbolic lines, and each square is the same hyperbolic size.

There is a *regular tessellation*, $\{p, q\}$, of the hyperbolic plane by regular p-sided polygons, q meeting at each vertex provided $(p-2)(q-2) > 4$. If $(p-2)(q-2) = 4$, one obtains one of the three Euclidean tessellations, the square grid $\{4, 4\}$, the hexagon grid $\{6, 3\}$, and the equilateral triangle grid $\{3, 6\}$. Vasarely made considerable use of the square grid, some use of the hexagon grid, but no use that I could find of the $\{3, 6\}$ grid. Figure 2 shows the regular tessellation $\{4, 6\}$, upon which Figure 1 is based. Figure 3 shows a pattern based on the $\{4, 5\}$ grid. In Figures 1 and 3 the colors were determined by formulas that varied radially and had 180° rotational symmetry. For more on creating patterns in the Poincaré model see [1].

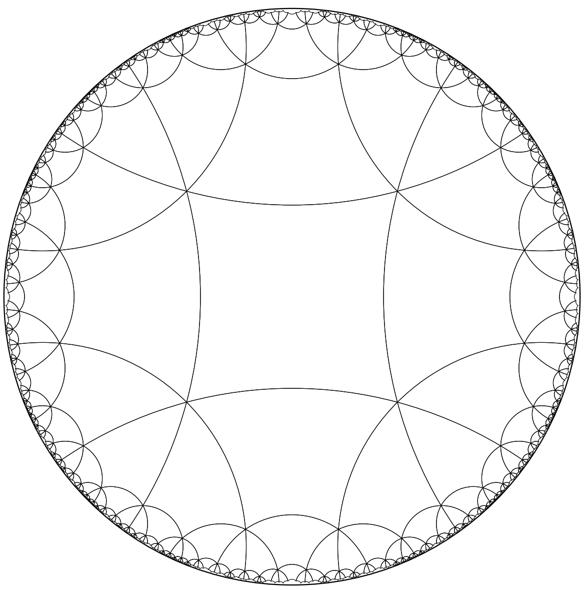

Figure 2: The {4,6} tessellation

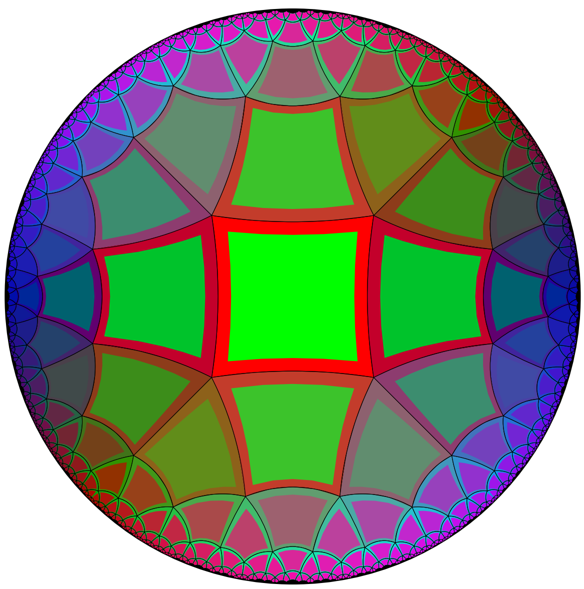

Figure 3: A pattern of squares based on the $\{4, 5\}$ tessellation.

4. Squares and Circles on Grids

As mentioned above, Vasarely created many patterns based on square grids, often with slightly smaller squares within the grid in a different color to produce squares with square "frames". In his *Vega* patterns, he distorted the grid to give the impression of a hemispherical bulge underneath, as shown in Figure 4[1]. Vasarely also placed circles within the squares of a *Vega* pattern, as in Figure 5. In these patterns, he smoothly varied the colorings of the squares and circles. In our patterns, we do not use any distortion other than that inherent in the Poincaré model, since that model is already unfamiliar enough. Figures 1 and 3 above show patterns in the framed square style of Vasarely, based on the hyperbolic grids {4,6} and {4,5} respectively. Figures 6 and 7 show patterns of circles on the respective hyperbolic grids {4,6} and {4,5}.

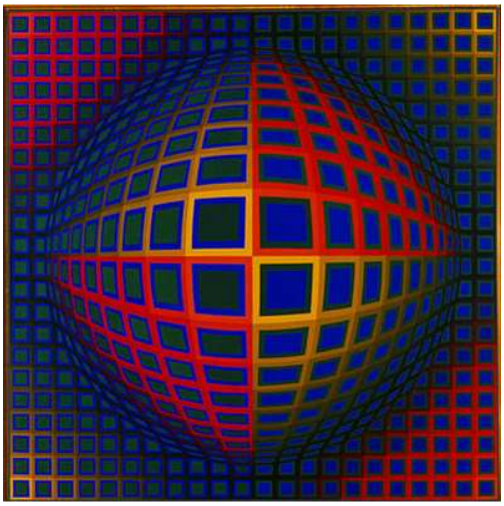

Figure 4: A distorted Vasarely *Vega* pattern of squares on a square grid.

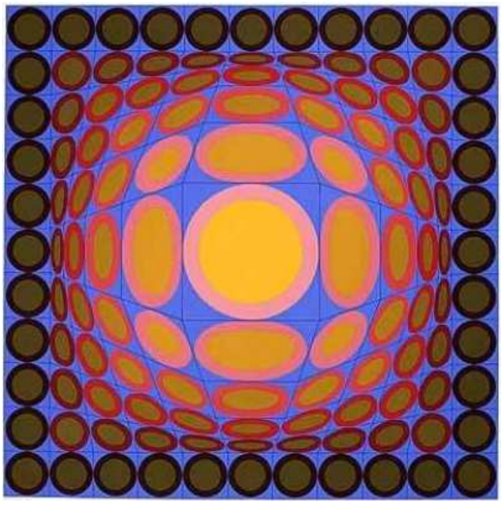

Figure 5: A Vasarely *Vega* pattern of circles on a square grid.

5. Patterns with Random Colors

In a pattern of squares, Vasarely sometimes chose the color for each square and its frame seemingly "randomly" — actually, I think the colors were very carefully chosen. Figure 8 shows such a pattern (with a cube-like distortion). Figure 9 shows a hyperbolic pattern in which the colors of the squares and frames were chosen by a random number generator. However, I liked the patterns of Figures 10 and 11 better, in which the colors of the frames are lighter versions of the colors of the interior squares.

Of course it is also possible to use randomly colored circles on a grid pattern. Randomly colored backgrounds seem to work better for circles than for squares. Figures 12 and 13 show two such randomly colored circle patterns.

6. Patterns Based on a Hexagon Grid

Vasarely also designed some patterns based on the hexagon grid {6,3}. He filled each hexagon with 3 rhombi, and colored them in such a way as to suggest the isometric projection of a 3D cube. Figure 14 shows such a pattern with distortion in the center. Figure 15 shows a hyperbolic version based on the {6,4} grid. Unfortunately the colors do not work as well, with some adjacent "rhombi" being colored the same or nearly the same. In this pattern each of the rhombi is colored by a different "light source": rhombi facing

[1]Figures 4, 5, 8, & 14 ©Victor Vasarely Official Artist Web Site `http://www.vasarely.com/`

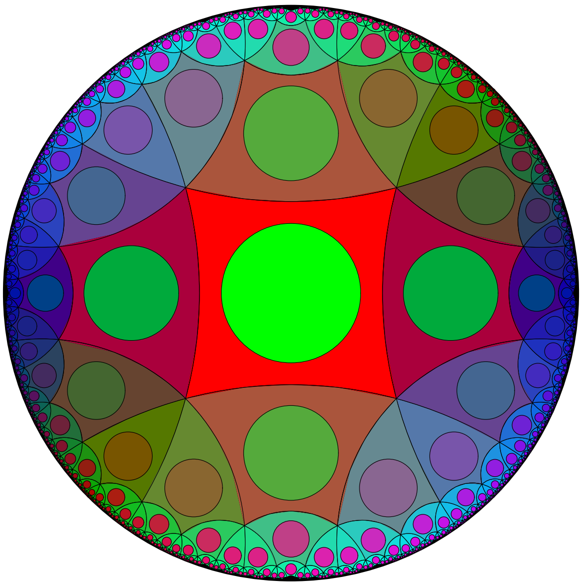

Figure 6: A hyperbolic pattern of circles on a {4,6} grid.

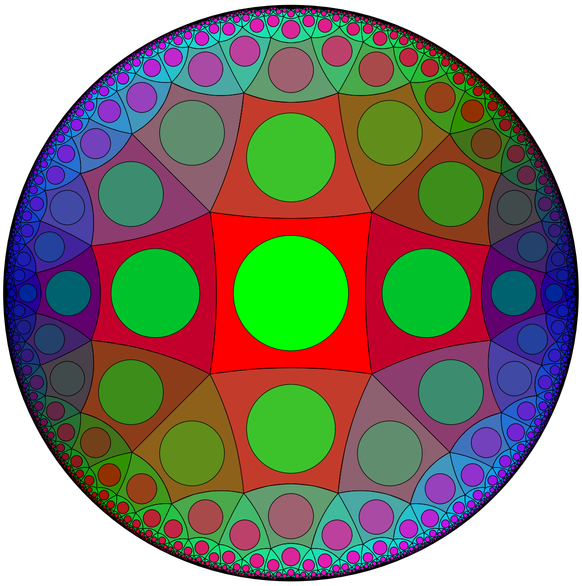

Figure 7: A hyperbolic pattern of circles on a {4,5} grid.

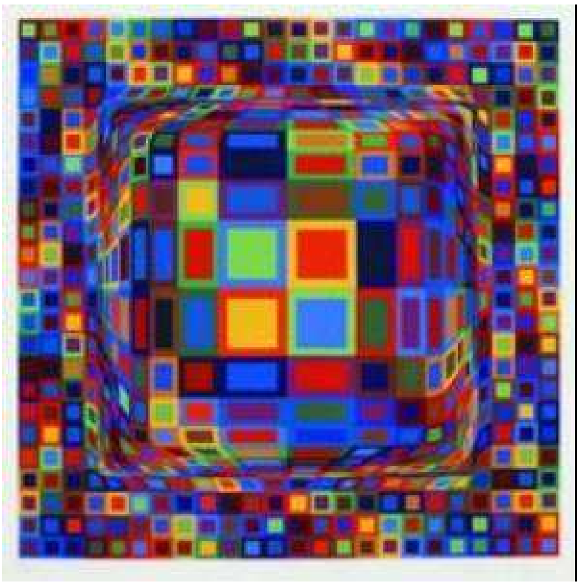

Figure 8: A Vasarely pattern of seemingly "randomly" colored squares.

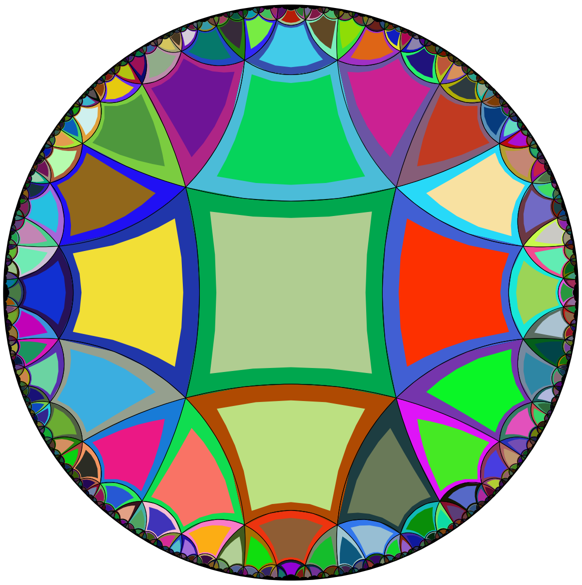

Figure 9: A hyperbolic pattern of randomly colored squares with contrasting frames.

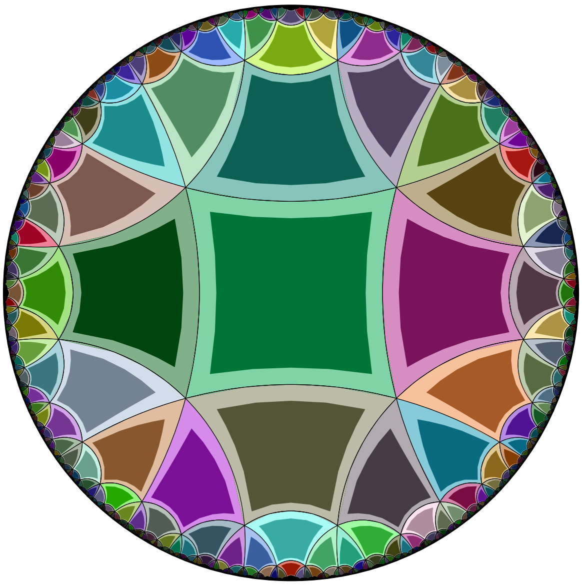

Figure 10: A pattern of randomly colored squares with lighter frames on a $\{4,6\}$ grid.

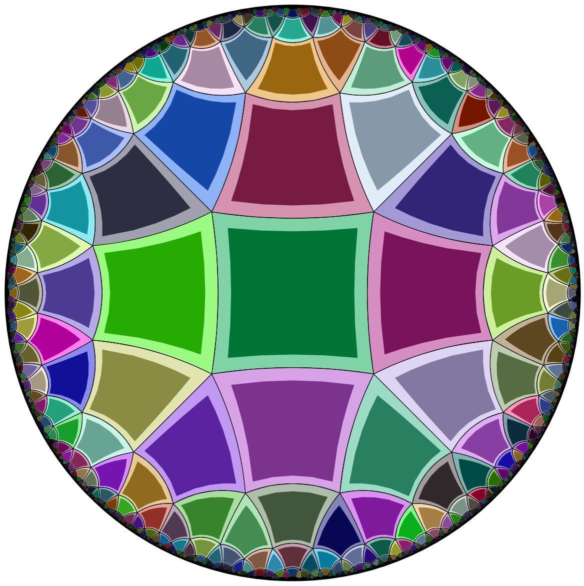

Figure 11: A pattern of randomly colored squares with lighter frames on a $\{4,5\}$ grid.

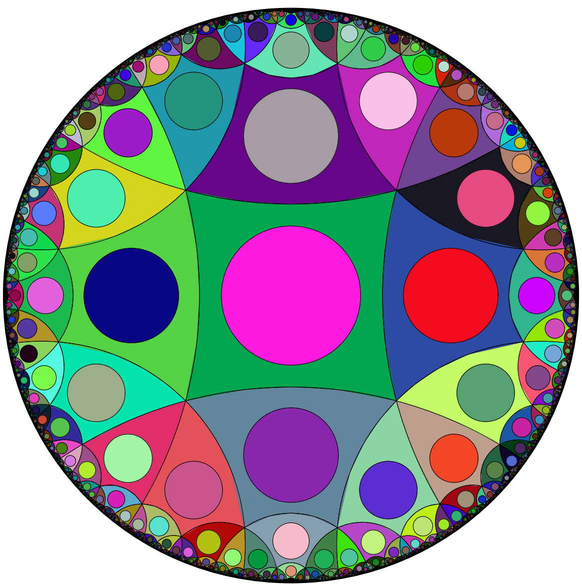

Figure 12: A pattern of randomly colored circles on a $\{4,6\}$ grid.

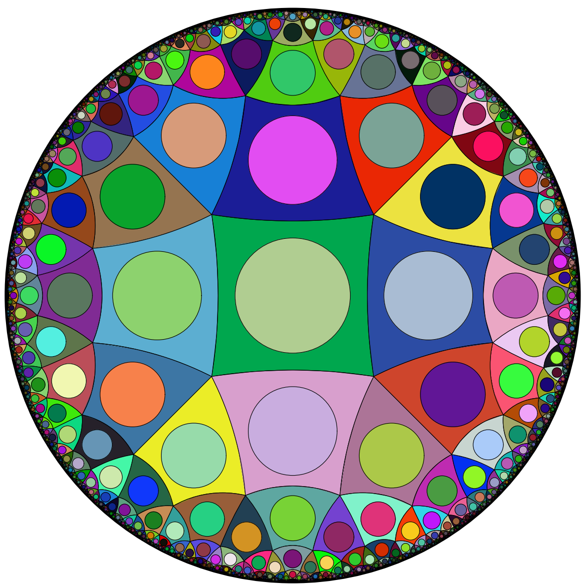

Figure 13: A pattern of randomly colored circles on a $\{4,5\}$ grid.

upward are colored by a yellow light, those facing to the left by a "gray" light, and those facing to the right by a "dark gray" light. There is some variation in the colors, since the closer the rhombi face to their light source, the brighter they are according to Lambert's cosine law for diffuse reflection.

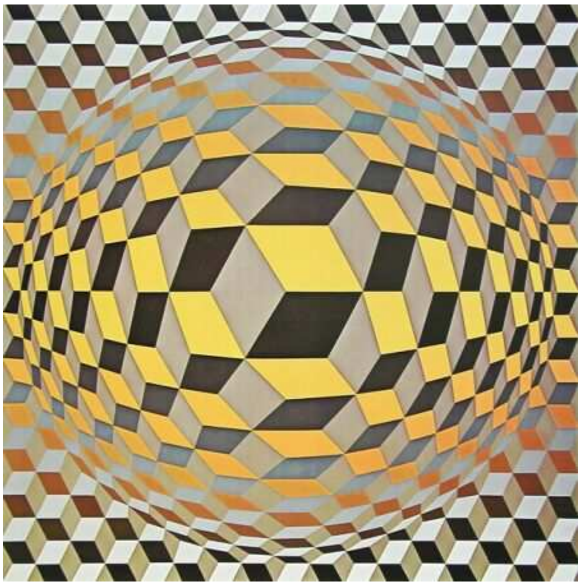

Figure 14: A Vasarely pattern based on a hexagon grid.

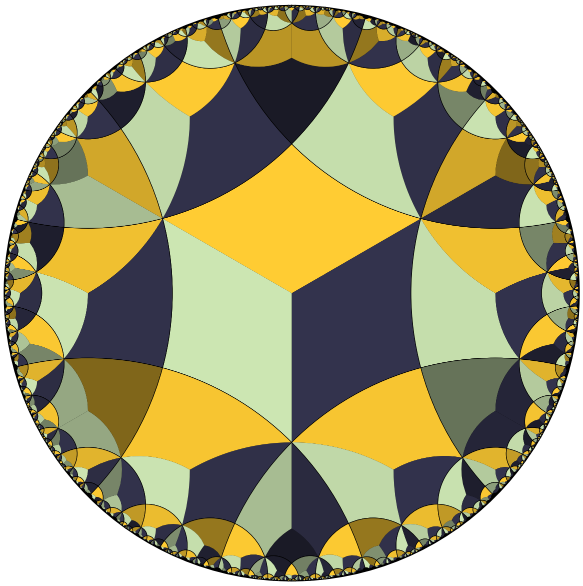

Figure 15: A "lighted" hexagon pattern based on a $\{6,4\}$ grid.

7. Conclusions and Future Work

We have designed some hyperbolic patterns based on works of Vasarely. However there are certainly many more possibilities to explore since he created so many geometric patterns.

As mentioned in the Introduction, I think the attempt at constructing a hyperbolic hexagon pattern was not as successful as the others. It would be nice to find a more satisfying hyperbolic example of a Vasarely-like hexagon pattern.

Acknowledgments

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References

[1] D. Dunham, Hyperbolic symmetry, *Computers & Mathematics with Applications*, Vol. 12**B**, Nos. 1/2, 1986, pp. 139–153. Also appears in the book *Symmetry* edited by István Hargittai, Pergamon Press, New York, 1986. ISBN 0-08-033986-7

[2] M. Vasarely, *Official Vasarely web site* at: `http://www.vasarely.com/`

[3] V. Vasarely, *Vasarely I, II, III, IV (Plastic Arts of the Twentieth Century)*, Editions du Griffon Neuchatel, 1965, 1971, 1974, 1979. ASIN's: B000FH4NZG, B0006CJHNI, B0007AHBLY, B0006E65FY