Outline

- Background and inspiration
- M.C. Escher's Metamorphosis prints
- Our metamorphosis images
- Conclusions and future work
- Contact information
Background and Inspiration

Our goal is to show patterns with different kinds transitions.

Most of our images start in an ordered way but end up in a less ordered, more chaotic state.

In contrast, M.C. Escher’s prints usually show transitions from one kind of order to another — a pattern based on squares deforming into a pattern based on hexagons, for example.

In fact, Escher’s three “Metamorphosis” woodcuts were part of the inspiration for our work. We show them below.

Craig Kaplan discussed Escher transformations in his 2008 Bridges paper *Metamorphosis in Escher’s Art.*
Escher’s Metamorphosis Woodcuts Work

- *Metamorphosis I*: 1937, about 36 by 7.7 inches.
- *Metamorphosis II*: 1940, about 13 feet by 7.7 inches.
- *Metamorphosis III*: 1967, about 22 feet by 7.7 inches.
Metamorphosis I
Metamorphosis II
Metamorphosis III
Our Metamorphosis Patterns

Order and Chaos
Related: “Gravel Stones” Georg Nees, 1966
Earlier: (Untitled) Jean Arp 1916–17
Ideas behind our patterns

In going from order to disorder, we can vary several quantities starting with regular values and ending with random values in some range. Some of those quantities are listed below.

- The positions and orientations of elements of the image.
- The boundaries of the elements — going from flat to wavy.
- The curvature of a path, starting out straight.
- The colors — often starting from black and white, then tracing a path through RGB space.
- Adding interior details to elements, starting with no such details.
Bandwidth-Limited Noise

One way to generate bandwidth-limited random values is to use a 2D Fourier series (which has the advantage that it is periodic). If $X$ and $Y$ are the width and height of the image, the value $f(x, y)$ can be given by:

$$f(x, y) = \sum_{i,j} A_{ij} \cos(k_i x + \phi_{xi}) \cos(k_j y + \phi_{yj})$$

where $k_i = 2\pi / X$ and $k_j = 2\pi j / Y$, the $A_{ij}$ are chosen randomly in the interval $[0, 1]$, the $\phi$’s are chosen randomly in the interval $[0, 2\pi]$, and $i$ and $j$ are integers in the range $[-M, M]$ where $M$ is usually less than 10. This produces “smooth” noise.

To generate noise with higher frequencies, it is more efficient to use Perlin noise. The pattern of the next slide was produced this way.
Entropy
“Autologlyphs” Henry Segerman, 2009
Day into Night
Teach a Fish to Swim
Squares to Hexagons
Progressive Butterflies I
Progressive Butterflies II
Two Birds — a radial transition
Out to In
Waves of Emotion — Chernoff faces?
Soldiers
Elves
Fade to Gray
Quantum Entanglement Explained
Future Work

- We have shown some transitioning patterns, but certainly there are more possibilities. We have not even explored all the Escher prints for more inspirations.

- We have shown some elements that can be varied, such as position, orientation, shape of the boundary, and color. But there are an unlimited number of functions to apply to achieve such variations, and we have barely begun to explore them.
Of course we owe Reza Sarhangi a tremendous debt for inspiring us to do mathematical art.

We would also like to thank the organizers of Bridges 2018.

Contact Information:
Doug Dunham
Email: ddunham@d.umn.edu
Web: http://www.d.umn.edu/~ddunham

John Shier
Email: johnpf99@frontiernet.net
Web: http://www.john-art.com/statgeom_linkpage.html