Embroidery of a Hyperbolic Fish Pattern

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Outline

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Inspirations

Our first inspiration was M.C. Escher’s four “Circle Limit” patterns, which motivated the first author, Dunham, to create a computer program that could replicate those patterns in 1980.

Secondly, in March, 2018 Dunham visited a weaving and embroidery shop in Varanasi, India and wondered if embroidery techniques could also create “Circle Limit” patterns.
M.C. Escher’s pattern Circle Limit II - Dunham’s version
M.C. Escher’s woodcut Circle Limit III
M.C. Escher’s woodcut Circle Limit IV
An ordinary loom in Varanasi, India
A Jacquard loom in Varanasi, India
We review the following topics:

- Hyperbolic geometry.
- The hyperbolic program can create other patterns in the pattern families of which Escher's *Circle Limits* are examples, in particular the pattern of the title slide.
- A family of *Circle Limit III* patterns.
Hyperbolic Geometry

- In 1901, David Hilbert proved that, unlike the sphere, there was no smooth isometric (distance-preserving) embedding of the hyperbolic plane into ordinary Euclidean 3-space.

- Thus we must use *models* of hyperbolic geometry in which Euclidean objects have hyperbolic meaning, and which must distort distance.

- One such model is the *Poincaré disk model*. The hyperbolic points in this model are represented by interior point of a Euclidean circle — the *bounding circle*. The hyperbolic lines are represented by (internal) circular arcs that are perpendicular to the bounding circle (with diameters as special cases).

- This model is appealing to artists since (1) angles have their Euclidean measure (i.e. it is conformal), so that motifs of a repeating pattern retain their approximate shape as they get smaller toward the edge of the bounding circle, and (2) it can display an entire pattern in a finite area.
Poincaré Disk Model of Hyperbolic Geometry
A Family of *Circle Limit III* Patterns

We use the symbolism \((p,q,r)\) to denote a pattern of fish in which \(p\) meet at right fin tips, \(q\) meet at left fin tips, and \(r\) fish meet at their noses. Of course \(p\) and \(q\) must be at least three, and \(r\) must be odd so that the fish swim head-to-tail (as they do in *Circle Limit III*).

Escher’s *Circle Limit III* pattern itself would be labeled \((4,3,3)\) in this notation.

We note that our \((5,3,3)\) pattern of the title slide requires six colors, whereas *Circle Limit III* only needs four. In fact the color symmetry group of our pattern is the alternating group \(A(5)\).
The (5,3,3) pattern of the title slide
Previous Hyperbolic Artworks the Fiber Arts

- 2D patterns using the Poincaré model
  - Tony Bomford’s hooked rugs
  - Mary Williams’ quilt
- 3D works
  - Diana Taimina’s crochet works
  - S. Louise Gould’s triply repeating polyhedra
Tony Bomford’s Rug 17
Mary Williams’ quilt “Poincaré”
Diana Taimina’s \(\{6,4\}\) surface
Louise and Frank Gould’s (5.5.5.5.5) surface
Our First Pattern

- Used a Viking Husqvarna Epic embroidery machine with a 260mm by 360mm hoop.
- And used Floriani Total Control embroidery software.
Husqvarna Epic embroidery machine
Floriani Total Control embroidery software
The pattern with only the fish bodies
The completed 10.5 inch pattern
Our Final Pattern

- Used a 350mm by 360mm Husqvarna Majestic hoop — the largest available.

- Advantages:
  - A bigger pattern! — 13.75 inches.
  - More fish — the fish pattern gets closer to the bounding circle.
  - More detail within the outer fish.

- Disadvantages:
  - It could only be used to stitch half of the pattern at a time.
  - The hoop must be rotated 180 degrees to stitch the second half — actually it had to be rotated many time to complete the pattern.
  - After rotation the hoop had to be carefully aligned.
Aligning the bounding circle
Aligning the fish outlines
The final pattern
The 10.5 inch pattern for comparison
Another Escher-inspired hyperbolic pattern

- Escher's Regular Division Drawing 42 of shells
- A related hyperbolic pattern of shells
A hyperbolic pattern of shells
Future Work

- There are many more hyperbolic circle patterns that could be embroidered, some more easily than others.

- And, inspired by fiber artists who have worked in 3D, we would like to embroider Escher-like patterns on 3D surfaces.

- We would also like to explore papercrafting a hyperbolic pattern with a digital cutter/printer.

- Another possibility is patterns on free standing lace.
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