Lesson Topic: **Dividing Polynomials**  
Length of lesson: **60 minutes**  
Grade level: **10th – 12th**

### Stage 1 – Desired Results

#### Content Standard(s):

**Algebra Standard:** Generate equivalent algebraic expressions involving polynomials and radicals; use algebraic properties to evaluate expressions.

- 9.2.3.2 Add, subtract and multiply polynomials; **divide a polynomial by a polynomial of equal or lower degree**.
- 9.2.3.4 Add, subtract, multiply, divide and simplify algebraic fractions.

#### Understanding(s)/Goals:

Students will understand:

- How to divide polynomials using long division and synthetic division. This can be related to the Cognitive Domain of Bloom’s Taxonomy (1956) at the Knowledge level since students are required to “know” and understand dividing polynomials according to the state standards. This would be considered part of the learned information that is a basis for future concepts to be built upon.
- How to apply algebraic concepts to solve real-world and mathematical problems. According to Newmann’s Authentic Pedagogy (1995), there needs to be some relevancy in a topic to a student’s life. When students can actually connect the material to the real world, they are more likely to remember what it is that they learned as well as the actual process they were engaged in during the learning.

#### Essential Question(s):

**For the unit:**

- Why would we study polynomials and their various operations? By asking a question that assigns value to the topic, the emotions and personal values of the students are related to the material, so according to Bloom’s Affective Domain (1956), students are more likely to remember the material.

**For the lesson:**

- Where do you see the use/application of dividing polynomials in the world around you? Why is this important? This question addresses the relevancy, Newmann’s Authentic Pedagogy (1995), as well as the value, Bloom’s Affective Domain (1956), of the topic. Having students determine where they see polynomials around them and why they are important, allows them to make a connection to the real-world and involve their personal values. Students will be more likely to remember the material since they are making these connections.
- What similarities and differences exist between multiplying and dividing polynomials? This question addresses both the Knowledge and Comprehension levels of Bloom’s Cognitive Domain (1956). Students have to use and understand their knowledge in order to make these connections.
**Student Objectives (Outcomes):**

**Cognitive Domain**
All students will be able to:
- Divide polynomials using long division and synthetic division. Since all students should be able to achieve this task, it would most likely fall into the Knowledge level of Bloom’s (1956) Taxonomy. This is because students should be able to recognize and recall the information the way it was presented.

Most students will be able to:
- Explain their thinking when working through the problems. This coincides with the Comprehension level of Bloom’s (1956) Taxonomy. Through the explanations of the students, they are interpreting and showing their grasp of the concepts.
- Describe what essential characteristics are needed of a division problem in order to use synthetic division. Another Comprehension level objective. Most of the students at this level should be able to distinguish which problems are capable of being solved using synthetic division and be able to explain what the deciding characteristics are in order to perform the computation.

Some students will be able to:
- Construct their own “Mind-Reading Trick” algorithm. Although most students will find the “Mind-Trick” amusing, only some of the students will be at Bloom’s (1956) Application level. That is, only some of the students will be able to use what they have learned in order to successfully create their own “Mind-Trick” algorithm.

**Psychomotor Domain**
All students will be able to:
- Follow the division algorithm and the process of synthetic division (with some fundamental errors in these processes). Since all students should be able to watch the teacher and then repeat these processes, this objective is at the Imitation level in Bloom’s (1956) and Dave’s (1967) Taxonomy. Although the students should be able to mimic the steps, some of the students might not necessarily understand what they are doing. Thus, there is more room for errors in the problems because

Most students will be able to:
- Perform the tasks needed in order to divide polynomials (with a few fundamental errors in the problems). Students at this level of Manipulation in Bloom’s (1956) and Dave’s (1967) Taxonomy should be able to carry out the task of dividing with more comprehension than some of the students in the “All students” category. That is, there should be less fundamental errors in the process because there is more understanding.

Some students will be able to:
- Demonstrate the processes to other learners in their partnerships as well as execute the skill of dividing polynomials reliably, independent of help (with little to no fundamental errors in the problems). Only some students will be able to be at the Precision level of Bloom’s (1956) and Dave’s (1967) Taxonomy. These would be some of the students who would be able to correctly solve the first few class examples during the exploration time before the teacher explanation. These students would be the ones in their partnerships guiding and showing their fellow classmates what they did in order to solve the problem and be able to justify their answer with their reasoning.
**Affective Domain**

All students will be able to:

- Follow the class examples as well as perform and practice the problems. This objective falls into the Receiving Phenomena and into the Responding to Phenomena levels of Bloom’s (1956) Taxonomy. All students should be able to listen to their peers with respect in their partnerships and be able to follow what is going on in class. Also, all students should be able to actively participate in the class discussions as well as perform and practice the division problems.

Most students will be able to:

- Differentiate when to use synthetic division instead of long division. By being able to differentiate when it is appropriate to use one division method over another, students are showing their ability to solve problems and are placing value on the particular procedures. Thus, this objective meets the criteria in the Valuing level of Bloom’s Taxonomy (1956). That is, most students will consistently be able to distinguish between which procedures to use.

- Justify their work and answers. Another example of an objective in the Valuing category in Bloom’s Affective Domain (1956). When students justify what they have done, it is having them place value on which processes they used and why they may have chosen to do one thing over another.

Some students will be able to:

- Formulate their own “Mind-Reading Trick” algorithm. Although this is also an objective that is part of the Cognitive domain, it is also very much a part of the Affective domain in Bloom’s Taxonomy (1956). This particular objective has students creating a unique value system of their own, which is a characteristic of the Organization level.

**Stage 2 – Assessment Evidence**

<table>
<thead>
<tr>
<th>Performance Task(s):</th>
<th>Other Evidence:</th>
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<tbody>
<tr>
<td>Students will have a choice between either of the following tasks:</td>
<td>Pre-assessment</td>
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<tr>
<td>- Select an area of interest that involves the division of polynomials (examples from the book include manufacturing, business, entertainment, medicine, physics, etc.) and explain how polynomials are used in these fields (give an example that is worked out by the student) and why polynomials are important to these fields. Students can choose how they want to present the information (essay, dialogue, etc.). A rubric will be used in assessing their work.</td>
<td>- Exploration of examples in their partnerships before the full explanation by the teacher. Students are constructing their own knowledge which is a key concept of Newmann’s Authentic Pedagogy (1995).</td>
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<tr>
<td></td>
<td>Formative Assessment (for learning)</td>
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<td>- Homework assignment Students will be practicing the procedures.</td>
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<td></td>
<td>- Classroom discussion Students will be helping each other as well as answering questions about procedures.</td>
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<td></td>
<td>Summative Assessment (of learning)</td>
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<tr>
<td></td>
<td>- Performance task The understanding of what the student has learned will be made apparent in the performance task.</td>
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</tbody>
</table>
- Construct a “Mind-Reading Trick” algorithm that is a lot more elaborate than the one used in class. For this, students must have the algorithm written out step by step (10 or more steps) and must also demonstrate how their algorithm works in a dialogue with the teacher. A rubric will be used in assessing their work. This performance task addresses the Cognitive domain of Bloom’s Taxonomy (1956) at the Application level. Students will use what they have learned in order to successfully create their own “Mind-Trick” algorithm. This is also a part of the Affective domain in Bloom’s Taxonomy (1956). This task has students creating a unique value system of their own, which is a characteristic of the Organization level.

- Classroom discussion Students will be describing how and why they did something.
- Homework assignment Students will have the opportunity to work on some problems that gets them to think more critically about why and how something is the way it is.

### Stage 3 – Learning Plan

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<th>Learning Activities:</th>
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<tbody>
<tr>
<td>Materials and Resources:</td>
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<tr>
<td>- <em>Algebra 2</em> Textbook</td>
</tr>
<tr>
<td>- Notebook or Paper</td>
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<td>- Pencil or Pen</td>
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<tr>
<td>- Overhead of “Mind-Reading Number Trick” Steps</td>
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<tr>
<td>- Rules for Synthetic Division handout</td>
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**Introduction: (8 minutes)**

- Once the students have settled into their seats, in order to engage their attention right away, start off the class period by using the “Mind-Reading Number Trick” with the class. By starting class right away with something to engage the attention of the students, the teacher is planning to prevent any potential classroom management problems before any can formulate. This is practicing Discipline with Dignity according to Curwin and Mendler (2007).

- Inform the students that they are allowed to use a calculator, pencil and paper, or they can try and perform the calculations in their heads (if they are choosing this last option they should probably choose a smaller number, unless they want a real challenge, so calculations are easier). By giving students options, it allows students to feel comfortable enough to attempt the task. This is using Choice Theory (Glasser, 2007) as well as Affordance Theory (Gibson, 1979). It also provides for opportunity for students to challenge themselves since having the three different options appeals to different learners. While performing the trick, make sure you are pausing after each step...
step so that all students have enough time to do their calculations. Have the students to select any positive number of their choice.

1. Tell the students to square their number (multiply the number by itself; that is, if the number is 5, the student would end up with $5 \times 5 = 5^2 = 25$).

2. Then, have the students add their result from the first step to their original number (for our example, it would be $25+5 = 30$).

3. Next, they need to divide their result from step two by their original number (so we would have $30 \div 5 = 6$).

4. After that, have the students give you any one number of their choosing (one number for the entire class) and tell them to add their result from step three to this number (say the class chooses 34, then it would be $6+34 = 40$).

5. Then, have the students subtract their original number from their result from step four (so we would have $40-5 = 35$).

6. Finally, tell the students to divide their result from step five by 5 (for our example, it would be $35 \div 5 = 7$).
   - What you tell the students to divide by depends on their choice in step 4. Whatever number is used in step 4, add 1 to it. This will give you the number you need in order to decide what to tell students to divide by. (See explanation in the conclusion.)

- After the sixth step, say, “I’ll bet that the number you are thinking of right now is 7,” and see what the students’ reactions are (surprised, not surprised, interested, etc.). Using the “Mind-Reading Trick” allows for an opportunity for students to respond with wonderment and awe, a valuable Habit of Mind according to Costa and Kallick (2000). It should raise the curiosity of most students and have them wondering how the trick was done. All of their attention will be focused on the teacher.

- Ask the students, “What was your original number?” Have some of the students share what their starting number was with the class. From this, students should see a wide range of starting numbers. This is using Practice Theory (Lippmann, 2007) since it calls for social interaction amongst the students and teacher in the class.

- “You are all probably wondering how even though everyone had different starting numbers, you all ended up with the answer 7. Does anyone have any guesses as to how I did this?” Allow for some possible explanations from the students. When students are participating in a discussion like this, they are developing Linguistic, Intrapersonal, and Interpersonal intelligences (Gardner, 1983). By having to communicate what they are thinking, they must first know it themselves and then be able to verbalize their thoughts to their classmates. In this particular context, students are also developing Logical-Mathematical intelligence. Also being cultivated through this questioning is the Habit of Mind of Thinking and Communicating with Clarity and Precision (Costa & Kallick, 2000). Then follow with, “We will leave this to be answered
later in the lesson, but I want all of you to be thinking about how this might be possible.”

Body: (45 minutes)

• “Yesterday in class, we learned about polynomials. Today we will be using that knowledge of polynomials, as well as what we know about monomials, in order to learn how to divide polynomials using two different methods.” In other words, students will be Applying Past Knowledge to New Situations, a valuable Habit of Mind (Costa & Kallick, 2000). This lesson is on Section 5-3 Dividing Polynomials on pages 233-236 in the textbook.

• Since the students have already learned how to divide monomials, have them use that knowledge to attempt to solve the first example (which should be written on the board or overhead for them to see) with partners. *Getting students to Think Interdependently is another valuable Habit of Mind (Costa & Kallick, 2000). This is also an example of Vygotsky’s (1978) theory regarding the Zone of Proximal Development (ZPD). Students are learning in relationship to others who might be more knowledgeable and might have more experience. Students are also attempting to construct their own knowledge since they are asked to try and work through the examples in their partnerships without teacher guidance. This is an example of Newmann’s Authentic Pedagogy (1995) in practice. Also, by allowing them to be in partners and collaborating on the work, it allows for learners, who like having the face-to-face time with people, to have the opportunity to work with others.* (2 minutes)

• While the students are working on the problem, walk around to see how they are doing and answer any questions they might have. *This is an example of using proximity control, as described in the Discipline with Dignity theory, as a prevention/action step in managing potential discipline problems (Curwin & Mendler, 2007).*

• Some students may not finish solving the example in the given time, which is alright. This is considered part of the informal pre-assessment of where students are in their understanding of the topic. Next, go over the steps to the problem on the board. Be asking the class to tell you what steps are needed in order to solve and be asking why/how they knew what steps to do. *Getting the students involved by having them describe what steps to perform next allows them to have a say in their own learning.*
It gives students some power while keeping them engaged and interactive, which are all components of Choice Theory (Glasser, 2007). This is also an example of the Responding and Valuing levels of Bloom’s Affective Domain (1956) and the Comprehension level of Bloom’s Cognitive Domain (1956) since students are asked to share their thought process and explain their work with their classmates. Students are also developing Linguistic, Intrapersonal, and Interpersonal intelligences (Gardner, 1983). By having to communicate what they are thinking, they must first know it themselves and then be able to verbalize their thoughts to their classmates. In this particular context, students are also developing Logical-Mathematical intelligence. Also being cultivated through this questioning is the Habit of Mind of Thinking and Communicating with Clarity and Precision (Costa & Kallick, 2000). This is also designed to help learners with differences. Visual and auditory listeners are being helped when the problem is worked through.* (5 minutes)

- Next, give the students the second example (on the board or overhead) and have them try and solve it with their partners. For theories that support this, please see * annotation above, just before Example 1. (3 minutes)

**Example 2**  

*Division Algorithm*

Use long division to find \((z^3 + 2z - 24) \div (z - 4)\).

\[
\begin{array}{c c c}
\multicolumn{3}{c}{z^2 + 2z - 6} \\
\hline
z - 4 & z^3 + 2z - 24 \\
- & \underline{z^2 - 4z} & \quad z^2 + 2z - 24 \\
\hline
& \frac{-4z}{6z - 24} & \quad z(z - 4) = z^2 - 4z \\
& - & \underline{\frac{-4z}{6z - 24}} \quad (\neg)z^2 - 4z \\
\hline
& 0 & \quad 6z - 24 \\
& & \quad (\neg)6z - 24
\end{array}
\]

The quotient is \(z + 6\). The remainder is 0.

- Walk around and answer any questions while students are working on the problem. For theories that support this, please see * annotation above, just after Example 1.

- Again, not all students will finish solving the problem. Also, this time more of the students probably will not solve it correctly. Go over the example with the class in the same manner as the first example (for theories that support this, please see * annotation above) and explain that this process is known as the *division algorithm*, which is similar to long division. At this point, you can ask the students, “What similarities and differences exist between multiplying and dividing polynomials?” Allow the students to discuss this with their partners and in the large group. This question addresses both the Knowledge and Comprehension levels of Bloom’s Cognitive Domain (1956). Students have to use and understand their knowledge in order to make these connections. So, students are also developing Intrapersonal, Interpersonal, and Linguistic intelligences (Gardner, 1983). (8 minutes)

- Next, have the students try the third example (which should be put up on the board or overhead) with their partners. For theories that support this, please see *
annotation above, just before Example 1. (This does not have to be done with the multiple choice format. The students can just be given the expression.) (2 minutes)

Example 3 Quotient with Remainder

Multiple-Choice Test Item

Which expression is equal to \((t^2 + 3t - 9)(5 - t)^{-1}\)?

A) \(t + 8 - \frac{31}{5 - t}\)  
B) \(-t - 8\)  
C) \(-t - 8 + \frac{31}{5 - t}\)  
D) \(-t - 8 - \frac{31}{5 - t}\)

Read the Test Item
Since the second factor has an exponent of \(-1\), this is a division problem.

\[ (t^2 + 3t - 9)(5 - t)^{-1} = \frac{t^2 + 3t - 9}{5 - t} \]

Solve the Test Item

\[
\begin{array}{c}
-t - 8 \\
-t + 5 \left( \frac{t^2 + 3t - 9}{5 - t} \right) \\
\frac{8t - 9}{(5 - t)8t - 40} \\
\frac{31}{31}
\end{array}
\]

For ease in dividing, rewrite \(5 - t\) as \(-t + 5\).

\(-t(-t + 5) = t^2 - 5t\)

\(3t(-50) = 8t\)

\(-8(-t + 5) = 8t - 40\)

\(-9 - (-40) = 31\)

The quotient is \(-t - 8\), and the remainder is 31. Therefore,

\[ (t^2 + 3t - 9)(5 - t)^{-1} = -t - 8 + \frac{31}{5 - t} \]

The answer is C.

• Walk around and answer any questions while students are working on the problem. For theories that support this, please see * annotation above, just after Example 1.

• Again, probably not all of the students will finish solving the problem. This time more of the students might do the division correctly, but may not know what to do with the remainder. Go over the example with the class. Ask them to tell you what steps you need to perform and ask them why they did what they did and how they knew what to do. For theories that support this, please see * annotation above. (7 minutes)

• The last two examples will be more lecture-based rather than the student-discovery-based first few examples. Just because the format is switching a little, it does not mean that students cannot still be actively involved in the learning process. As the teacher works through the next two examples, they can continue to ask questions and have students hypothesize about what steps they should perform next. That way, students are still being actively involved in the class and are feeling valuable for being able to participate in the lecture/discussion. Choice (Glasser, 2007), Practice (Lippman, 2007) and Affordance Theory (Gibson, 1979) all address these issues. Explain to students that another method that can be used for dividing polynomials is

...
**Synthetic Division** is a simpler process for dividing a polynomial by a binomial. Suppose you want to divide \(5x^3 - 13x^2 + 10x - 8\) by \(x - 2\) using long division. Compare the coefficients in this division with those in Example 4.

\[
\begin{array}{c|cccc}
 & 5x^2 & -3x & +4 \\
\hline
x-2 & 5x^3 & -13x^2 & +10x & -8 \\
 & \underline{-5x^3 +10x^2} & & & \\
 & & -3x^2 & +10x & \\
 & & \underline{-(-3x^2+6x)} & & \\
 & & & 4x & -8 \\
 & & & \underline{-4x+8} & \\
 & & & & 0
\end{array}
\]

**Example 4 Synthestic Division**

Use synthetic division to find \((5x^3 - 13x^2 + 10x - 8)/(x - 2)\).

**Step 1**
Write the terms of the dividend so that the degrees of the terms are in descending order. Then write just the coefficients as shown at the right.

\[
\begin{array}{c|cccc}
 & 5 & -13 & 10 & -8 \\
\hline
2 & 5 & \underline{-10} & & \\
 & 5 & & & \\
\end{array}
\]

**Step 2**
Write the constant \(r\) of the divisor \(x - r\) to the left. In this case, \(r = 2\), bring the first coefficient, 5, down as shown.

\[
\begin{array}{c|cccc}
 & 2 & | & 5 & -13 & 10 & -8 \\
\hline
 & 5 & & \underline{-10} & & \\
 & 5 & & & & \\
\end{array}
\]

**Step 3**
Multiply the first coefficient by \(r\); \(2 \cdot 5 = 10\). Write the product under the second coefficient. Then add the product and the second coefficient: \(-13 + 10 = -3\).

\[
\begin{array}{c|cccc}
 & 2 & | & 5 & -13 & 10 & -8 \\
\hline
 & 5 & & \underline{-10} & & \\
 & 5 & & & & \\
\end{array}
\]

**Step 4**
Multiply the sum, \(-3\), by \(r\); \(2 \cdot -3 = -6\). Write the product under the next coefficient and add: \(10 + (-6) = 4\).

\[
\begin{array}{c|cccc}
 & 2 & | & 5 & -13 & 10 & -8 \\
\hline
 & 5 & & \underline{-10} & & \\
 & 5 & & \underline{-6} & & \\
 & 5 & & & & \\
\end{array}
\]

**Step 5**
Multiply the sum, 4, by \(r\); \(2 \cdot 4 = 8\). Write the product under the next coefficient and add: \(-8 + 8 = 0\). The remainder is 0.

The numbers along the bottom row are the coefficients of the quotient. Start with the power of \(x\) that is one less than the degree of the dividend. Thus, the quotient is \(5x^2 - 3x + 4\).

To use synthetic division, the divisor must be of the form \(x - r\). If the coefficient of \(x\) in a divisor is not 1, you can rewrite the divisor expression so that you can use synthetic division.

**Example 5 Divisor with First Coefficient Other than 1**

Use synthetic division to find \((8x^4 - 4x^2 + x + 4)/(2x + 1)\).

Use division to rewrite the divisor so it has a first coefficient of 1.

\[
\frac{8x^4 - 4x^2 + x + 4}{2x + 1} = \frac{(8x^4 - 4x^2 + x + 4) + 2}{(2x + 1) + 2}
\]

Divide numerator and denominator by 2.

\[
\frac{4x^4 - 2x^2 + \frac{1}{2}x + 2}{x + \frac{1}{2}}
\]

Simply the numerator and denominator.

Since the numerator does not have an \(x^3\)-term, use a coefficient of 0 for \(x^3\).

\(x - r = x + \frac{1}{2}\), so \(r = -\frac{1}{2}\).
• After finishing the last example, explain to students that all of these examples are similar to the problems that they will be working on for tomorrow’s homework.

• Ask students what some of the important rules are for using synthetic division. Allow time for some answers. This is an example of the Comprehension level of Bloom’s Cognitive Domain (1956) since students would show their understanding by contributing various rules they found to be important. It could also be seen as an example of the Organization level of Bloom’s Affective Domain (1956) since students are identifying the important rules that need to be remembered. Be looking for answers on the Rules for Synthetic Division handout. Then pass out the handout after all of the points have been discussed. (3 minutes)

Conclusion: (7 minutes)

• “Now that you know more about dividing polynomials, does anyone have any insights into what I may have done for the ‘Mind-Reading Number Trick’?” Wait for student responses before explaining the math behind the trick. This time allows students to use the knowledge they just learned in class in order to hypothesize various reasons for how the ‘Mind-Reading Trick’ worked. This is Applying Past Knowledge to New Situations, a Habit of Mind (Costa & Kallick, 2000) as well as the Knowledge, Comprehension, Application, and even possibly the Analysis level of Bloom’s Cognitive
Domain (1956). Explanation of the trick is as follows:

- Using the same steps (at this point, you can put up the steps on the overhead with the values filled in), instead of using a number, use the variable $x$ and work through the steps.

- First, $x$ is squared, so we end up with $x^2$.

- Second, we add the result to the original number so we get $x^2 + x$.

- Then, since we divide by the original number we have $(x^2+x) ÷ x = x + 1$.

- Next, we added the 34 to the result, by suggestion of the class, so we get $x + 1 + 34 = x + 35$. (Knowing that from the previous step we have to add 1 to whatever number is given by the class is crucial in telling students what to divide by in the last step.)

- This next step has us subtract our original number from the result in the previous step, so we end up with $x + 35 - x = 35$. So we can see that in this step, the original number is eliminated.

- So for the last step, you could have students divide by 5 or 7 in order for the class to all get the same answer.

- Explain to students that this is just one example of the use of polynomials and dividing polynomials in the world around them. Tell them to start thinking of other examples in the world that use polynomials since they will need to have an idea of this as part of the performance task that will be discussed in class tomorrow. That is, you will be touching on the question, “Where do you see the use/application of dividing polynomials in the world around you? Why is this important?” Here is making the connection to the real-world. This question addresses the relevancy, Newmann’s Authentic Pedagogy (1995), as well as the value, Bloom’s Affective Domain (1956), of the topic. Having students determine where they see polynomials around them and why they are important, allows them to make a connection to the real-world and involve their personal values. Students will be more likely to remember the material since they are making these connections.

- Students will now be allowed to have the rest of the class period to start on the homework assignment. The homework will be the Concept Check on P.236 #1-3 under the Check for Understanding as well as P.236-237 #16-48 (every other even), 52, and 58.
Student Handout

Rules for Synthetic Division

- To use synthetic division, the divisor must be of the form $x - r$.
  - The coefficient on $x$ must be 1.
  - $x$ is only raised to the first power.

- Write the terms of the dividend so that the degrees of the terms are in descending order.
  - If the numerator does not have a $x^n$-term, where $n=1, 2, 3, \ldots$ then use a coefficient of 0 for that term.

- The numbers along the bottom row are the coefficients for the quotient. To write out the answer, start with the power of $x$ that is one less than the degree of the dividend.

To see a more thorough step-by-step example, please see pages 234-235 in the textbook.
Overhead of “Mind-Reading Number Trick” Steps

Choose a positive integer, then:

1. Square it.

2. Add the result to your original number.

3. Divide by your original number.

4. Add _____ to the result from the previous step.

5. Subtract your original number.

6. Divide by ____.

The number you’re thinking of is ____!
Textbook Pages of Homework Assignment

Check for Understanding

Concept Check

1. OPEN ENDED Write a quotient of two polynomials such that the remainder is 5.

2. Explain why synthetic division cannot be used to simplify \[ \frac{x^3 - 3x + 1}{x^2 + 1} \]

3. FIND THE ERROR Shelly and Jorge are dividing \( x^3 - 2x^2 + x - 3 \) by \( x - 4 \).

<table>
<thead>
<tr>
<th></th>
<th>Shelly</th>
<th>Jorge</th>
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<tbody>
<tr>
<td>4</td>
<td>1 -2 1 -5</td>
<td>4 8 36</td>
</tr>
<tr>
<td></td>
<td>1 -6 25 -103</td>
<td>1 2 0 35</td>
</tr>
</tbody>
</table>

Who is correct? Explain your reasoning.

Practice and Apply

Simplify.

15. \( \frac{9xy^2 - 18x^2y}{3xy} \)

16. \( \frac{5x^2y - 6y^2 + 3x^2y}{xy} \)

17. \( 28c^3d - 42ac^2d + 56cd^3 \) \( + (14cd) \)

18. \( (12mn^3 + 9m^2n^2 - 15m^2n) + (3mn) \)

19. \( 2y^2z + 4y^2z^2 - 8y^2z^3(yz)^{-1} \)

20. \( (a^3b^2 - a^2b + 2a)(-ab)^{-1} \)

21. \( (b^3 + 8b^3 - 20b) \div (b - 2) \)

22. \( (x^2 - 12x - 45) \div (x + 3) \)

23. \( (n^3 + 2n^2 - 5n + 12) \div (n + 4) \)

24. \( (2c^3 - 3c^2 + 3c - 4) \div (c - 2) \)

25. \( (x^3 - 3x^2 + x - 5) \div (x + 2) \)

26. \( (6w^5 - 18w^3 - 120) \div (w - 2) \)

27. \( (x^3 - 4x^2) \div (x - 4) \)

28. \( (x^3 - 27) \div (x - 3) \)

29. \( y^3 + 3y^2 - 5y - 4 \)

30. \( m^3 + 3m^2 - 7m - 21 \)

31. \( y + 4 \)

32. \( m + 3 \)

33. \( x^3 - 7x^2 + x + 1 \)

34. \( 3x^5 + 5x^4 + 5x + 1 \)

35. \( x - 3 \)

36. \( (2x^3 + 3 + 2b + 3)(x + 1) \)

37. \( (x^3 + 32)(y + 2)^{-1} \)

38. \( (2x^3 - 5x^2 + 22x) \div (2x + 3) \)

39. \( (2x^3 - 4x^2 - 3x - 1) \)

40. \( 4x^3 + 5x^2 - 3x + 1 \)

41. \( 3x^2 + 5x^4 + 3x + 1 \)

42. \( 6x^4 + x^2 - 3x + 1 \)

43. \( 2x^4 + 3x^2 - 4x^2 - x - 6 \)

44. \( x^4 + x^2 - 3x + 1 \)

45. \( x^2 + 3x^2 + x - 3 \)

46. \( x^4 + x^2 - 3x + 1 \)

47. \( x^3 + 3x^2 + 3x + 2 \)

48. \( x^3 - 4x^2 + 5x - 6 \)

49. \( x^2 - x + 2 \)
49. What is $x^3 - 2x^2 + 4x - 3$ divided by $x - 1$?

50. Divide $2y^3 + y^2 - 5y + 2$ by $y + 2$.

51. **BUSINESS** A company estimates that it costs $0.03x^2 + 4x + 1000$ dollars to produce $x$ units of a product. Find an expression for the average cost per unit.

52. **ENTERTAINMENT** A magician gives these instructions to a volunteer:
   - Choose a number and multiply it by 3.
   - Then add the sum of your number and 8 to the product you found.
   - Now divide by the sum of your number and 2
   What number will the volunteer always have at the end? Explain.

**MEDICINE** For Exercises 53 and 54, use the following information.
The number of students at a large high school who will catch the flu during an outbreak can be estimated by $n = \frac{1700t^2}{t^2 + 1}$, where $t$ is the number of weeks from the beginning of the epidemic and $n$ is the number of ill people.

53. Perform the division indicated by $\frac{1700t^2}{t^2 + 1}$.

54. Use the formula to estimate how many people will become ill during the first week.

**PHYSICS** For Exercises 55–57, suppose an object moves in a straight line so that after $t$ seconds, it is $t^2 + t^2 + 6t$ feet from its starting point.

55. Find the distance the object travels between the times $t = 2$ and $t = x$.

56. How much time elapses between $t = 2$ and $t = x$?

57. Find a simplified expression for the average speed of the object between times $t = 2$ and $t = x$.

58. **CRITICAL THINKING** Suppose the result of dividing one polynomial by another is $x^2 - 6x + 9 = \frac{1}{x - 3}$. What two polynomials might have been divided?
Reflection

1. What basis or bases drove the decisions you made on this curriculum project.
   I based my lesson plan on Section 5-3 Dividing Polynomials in the *Algebra 2* textbook that is used at Central High School. When I sat down with my cooperating teacher, she informed me that we would be starting Chapter 5 when I would be starting in the school for the four weeks of my apprenticeship. I really wanted to write my lesson plan based on one of the sections in the chapter because I knew it would be very challenging to write a lesson plan that would be interesting and interactive based on the content. It would have been much easier for me to choose a fun and exciting topic to create my lesson, but I wanted to really stretch and try and make algebra (something I really enjoy) more enjoyable for the students. I did my best in trying to think “outside-of-the-box” in planning this lesson and I really hope that it shows. There are still some concepts in math that cannot be “discovered” by students, especially if it is a process like synthetic division. So, I had to revert to a more lecture-based setting, but I still tried to incorporate as much student interaction as possible.

2. What assumptions about teaching and learning underlie your decisions?
   Teachers need to have fun and show enthusiasm, but at the same time they need to be realistic about what strategies they can and cannot use when teaching particular content. Teaching a good, interesting lesson definitely involves a lot of work and it makes me understand why some teachers may not want to put in the effort of doing that anymore. It tends to be easier to just tell students what they need to know and leave it at that, but then we are not being the thoughtful professionals we should be. Trying to take into consideration the different types of learners was also very challenging. I do not feel that I was able to accomplish that as thoroughly as I would have liked to, but I felt restricted by the content. I honestly had a very hard time trying to think of various ways of teaching the content so that it would appeal to all types of learners. I feel that I was able to achieve this to some extent, but not nearly where I would like to be.

3. How are your decisions likely to affect student learning?
   My decisions as an educator are going to have a major impact on student learning. What I do will either inspire or turn off students to the subject and to learning in general. Planning a lesson that is exciting, relevant to the students’ lives, gets across all of the content that needs to be covered, and tries to incorporate options and adaptations for all learners is extremely challenging, but these are all things that had an affect on student learning and need to be considered.

4. Is your project written at a level which reflects readiness to be scrutinized by exemplar professionals at your apprenticeship site? Explain why or why not. If not, what resources and support would be useful in developing your work to the level at which it is ready for professional scrutiny?
   I do feel that my project is written at a level which reflects readiness to be scrutinized by exemplar professionals at my apprenticeship site. From what I have heard about the teachers, who all seem to be good teachers, the main teaching strategy that is used is lecturing. I feel that my lesson would appeal to that side of the teachers as well as challenge them to think “outside-of-the-box” about some of the ways they might be able to differentiate the method used to convey knowledge to students. I feel that I would be ready to show my lesson plan to the teachers and support my decisions for planning and designing it the way I
did. I would welcome constructive criticism since they have been teaching for so long and know what does and does not work, but I would still be willing to try out my lesson in my own time and make adjustments later as needed. Overall, I feel pretty confident about the lesson plan I created.

5. What else, if anything, should your reader know about this project?

I worked very hard on designing the actual lesson plan with the different theories from educational psychology in mind. Like I mentioned earlier, it would have been a lot easier for me to choose an easier topic that is more exciting and relevant to students, but I wanted to be challenged in trying to come up with a good lesson on dividing polynomials.