# Graphical Analysis of Free-fall Motion

**Goal:** To find the acceleration of an object in free-fall by using graphical techniques.

## Lab Preparation

To prepare for this lab you will want to review position vs. time graphs and velocity vs. time graphs for objects that are in free-fall. Knowing what the general shapes represent and knowing what the slopes of the graphs represent will make the lab much more understandable.

One of the equations of motion that we can use to help understand free-fall is:

$$\Delta y = v_o t + \frac{1}{2} a t^2$$

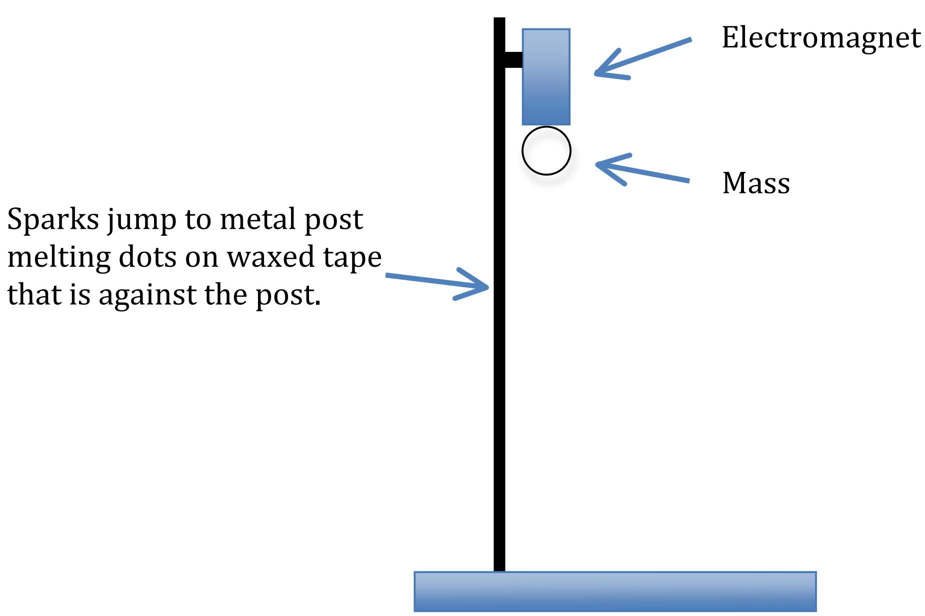
For this lab it is convenient to choose downward as positive. Thus $a = +g$ in this case. If we set $y_o = 0$ at $t = 0$ our equation simplifies to:

$$y = v_o t + \frac{1}{2} g t^2$$

## Equipment

For this experiment the lab instructor will operate the equipment. The instructor will use a free-fall apparatus that has a mass held in place by an electromagnet. When the current is turned off to the electromagnet, the mass falls to the ground.

To record the position of the mass as it falls, a spark timer is used. As the mass falls sparks are generated every 1/60 of a second and these sparks are recorded on waxed tape that is in place vertically next to the mass.



## Procedure

I. Recording data from waxed tape

Obtain a wax tape from your TA and tape it flat to the table for measurement. Examine the dots on the tape. You should see that they continually get further apart due to the acceleration of the mass as it falls. Occasionally a spark fails to form at the proper time and a dot will be missing. If you find this is true with your tape, read the (*) below.

To analyze the motion you will need to find the positions of all of the dots using a meter stick. The meter stick should be placed on its narrow edge so the rulings extend to the tape for easy accurate measurement. Line up a convenient spot on the meter stick with one of the first dots. Let this be $y_0 = 0$, $t = 0$. Make sure that the meter stick does not move from this point on until the table on your worksheet is completed. (Note: Just because we set $y_0 = 0$ at $t = 0$ doesn't mean the initial velocity is zero. Chances are the mass was moving before the first spark actually hit the paper.)

Since the dots are produced every 1/60 of a second, the time interval between every third dot is 3/60 s = .050 s. Record in your table the $t$ and $y$ values for every third dot.

(*) If you have one or two dots missing on a tape it is usually not a problem. Just pick a dot near the start of the tape so that the missing or questionable dot is one of the dots skipped over in counting out every third dot for measurement.

II. Graphing

Open the "freefall" file and enter your $t$, $y$ data in the designated columns. Examine the resulting graph of $y$ vs. $t$ to see if it seems reasonable. Make sure the limits of your graph include the origin (0,0). You can change the upper and lower limits on the axes by clicking on their values in the graph. Print out a copy of the graph (under "file" choose "page setup" and click on "landscape," then under "file" choose "print graph").

III. Linearization of the model equation

According to our model $y = v_o t + ½\, g t^2$ (with $y_o = 0$ at $t = 0$), a graph of $y$ vs. $t$ is expected to curve upwards. Data forming a straight line is easily recognized on a graph far more readily than determining if data correctly follow a particular curve. Thus, if we linearize our model equation it will provide a better way of analyzing the data.

Finding the best way to linearize an equation usually involves some algebraic manipulation of the model equation. For this case, it can be achieved by starting with the equation and simply dividing both sides by t:

$$y = ½\, g t^2 + v_o t$$

$$y/t = (½\, g)\, t + v_o$$

The standard form of an equation describing a straight line with slope $m$ and $Y$-intercept $b$ is: $Y = mX + b$. Comparing the standard form to our equation we can make the associations $Y = y/t$ and $X = t$.

We can also relate the new slope and intercept constants to the slope and intercept $m = ½\, g$ and $b = v_o$.

The new 'compound variable' $Y = y/\mathrm{t}$ is not a position – we are merely calling it $Y$ because it will form the vertical coordinate on a new graph. The new horizontal coordinate, $X$, in this case is just $t$.

Given the measured $(t,y)$ data, new data $(X,Y) = (t,\, y/t)$ can be calculated and plotted. A graph of $y/t$ vs. $t$ should produce a straight line if the acceleration is constant. The slope of this line should then be $m = g/2$ and the $Y$-intercept, $b$ should provide the initial velocity $v_o$.

Graphing your linearized model.
To plot $(X,Y) = (t,\, y/t)$ we will need to create a new column with $y/t$. You can create additional columns in the data table from the menu bar. Under "data" choose "New calculated column." Put in $y/t$ for the column name and short name, and enter the correct units in the appropriate box. In the expression box you can enter your formula $y/t$. There are two different ways you can do this. One is to choose the variables under the "variables" tab and put the division symbol (/) between them. The other way is to directly enter "*y*"/"*t*".

Go to page 2 and double click the graph. You should be able to select $y/t$ for the $y$-axis on your graph. Examine your graph. Is it a straight line? Adjust the range of both horizontal and vertical axes to include the origin (0,0). Adjust the maximum for both axes to round values that produce a neat looking grid. Once the graph is correctly prepared (suitable ranges, labels, units, title) print the graph but do not close the program.

IV. Analysis and Questions

1. On your $y/t$ vs. $t$ graph draw with a straight edge a straight line that best passes through the data. Extend the line you draw to the boundaries of the graph. Find the coordinates of the two points where your line intersects the graph's edges. Use the "Finding slopes accurately" pages to help you do this.

2. Find the slope and Y-intercept of your line based on the coordinates found above. Show your calculation clearly and with proper units.

3. Calculate the magnitude of the acceleration of gravity $g$ using the slope from your graph.

4. Find the percentage difference between your result and the local acceleration of gravity, 9.80 $m/s^2$.

$$\% \text{ difference} = \frac{|g - 9.80 \text{ m/s}^2|}{9.80 \text{ m/s}^2} \times 100\%$$

5. What is the initial velocity $v_o$? Explain why the initial velocity is not zero.

*When finished please clean up your table.

**Homework**

Sketch the $y$ vs. $t$, $v$ vs. $t$, and $a$ vs. $t$ graphs (just the general shapes) for an object that is dropped from rest and is in free fall. Let upward be the positive direction and let the bottom of the fall be y = 0. What value does the slope of the velocity vs. time graph for an object in free fall give (put answer next to your $v$ vs. $t$ graph)?