

Example Problem: 3.55 kg of air at a pressure of 101.3 kPa (abs) is confined by a piston/cylinder device with a volume of 3 m³. The piston radius is 0.9 m and atmospheric pressure, $P_{\text{atm}} = 101.3 \text{ kPa (abs)}$, acts on the external surfaces of the piston/cylinder. Determine the following:

- The density in kg/m³ and temperature in K of the air.
- The density in kg/m³ and pressure in kPa (abs) of the air if it is isothermally compressed by the piston to a volume of 1.5 m³.
- The force in N applied to the piston to maintain the 1.5 m³ volume of compressed air.

Given: 3.55 kg of air is isothermally compressed from state 1 to state 2 by a piston/cylinder.

$$m_1 := 3.55 \text{ kg}$$

$$\text{kPa} := 1000 \text{ Pa}$$

$$V_1 := 3 \text{ m}^3$$

$$V_2 := 1.5 \text{ m}^3$$

$$P_1 := 101.3 \text{ kPa}$$

$$T_2 = T_1 \text{ (isothermal)}$$

$$r_p := 0.9 \text{ m} \quad (\text{piston radius})$$

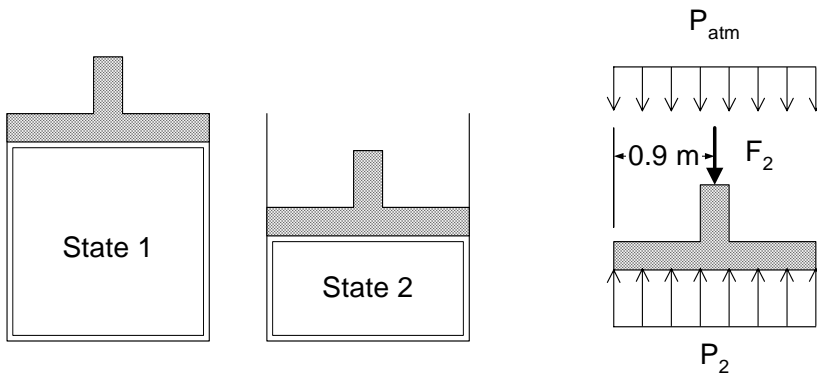
$$P_{\text{atm}} := 101.3 \text{ kPa}$$

Find: a) ρ_1 in kg/m³, T_1 in K

b) ρ_2 in kg/m³, P_2 in kPa (abs)

c) F_2 (force applied to piston at state 2 in N)

Schematic:



Assumptions:

1. No air leakage from piston/cylinder, therefore $m_1 = m_2$
2. Air is an Ideal Gas
3. No friction between piston and cylinder
4. Piston is "weightless"

Properties:

$$R_{\text{air}} := 286.9 \frac{\text{J}}{\text{kg} \cdot \text{K}}$$

from Table 1.8 front cover

$$k_{\text{air}} := 1.4$$

Solution:

a) density = mass/volume, $\rho := \frac{\text{mass}}{\text{volume}}$

$$\rho_1 := \frac{m_1}{V_1}$$

$$\rho_1 = 1.183 \frac{\text{kg}}{\text{m}^3}$$

Ideal Gas, eqn 1.8, p. 14

$$P := \rho \cdot R \cdot T \quad \text{therefore} \quad T := \frac{P}{\rho \cdot R}$$

$$T_1 := \frac{P_1}{\rho_1 \cdot R_{\text{air}}} \quad T_1 = 298.38 \text{ K}$$

b) isothermal compression of an ideal gas

Eqn 1.14, p. 23

$$\frac{P}{\rho} := \text{constant}$$

$$m_2 := m_1$$

$$m_1 = 3.55 \text{ kg}$$

$$\rho_2 := \frac{m_2}{V_2}$$

$$\rho_2 = 2.367 \frac{\text{kg}}{\text{m}^3}$$

$$\frac{P_2}{\rho_2} := \frac{P_1}{\rho_1}$$

therefore

$$P_2 := P_1 \cdot \frac{\rho_2}{\rho_1}$$

$$P_2 = 202.6 \text{ kPa}$$

$$\text{c) } \sum F := 0 \quad 0 := P_2 \cdot A - P_{\text{atm}} \cdot A - F_2$$

$$A := \pi \cdot r_p^2 \quad \text{Area of piston}$$

$$\text{therefore } F_2 := (P_2 - P_{\text{atm}}) \cdot \pi \cdot r_p^2$$

$$F_2 = 2.578 \times 10^5 \text{ N}$$