**Example Problem:** 3.55 kg of air at a pressure of 101.3 kPa (abs) is confined by a piston/cylinder device with a volume of 3 m³. The piston radius is 0.9 m and atmospheric pressure, $P_{\text{atm}} = 101.3$ kPa (abs), acts on the external surfaces of the piston/cylinder. Determine the following:

a) The density in kg/m³ and temperature in K of the air.

b) The density in kg/m³ and pressure in kPa (abs) of the air if it is isothermally compressed by the piston to a volume of 1.5 m³.

c) The force in N applied to the piston to maintain the 1.5 m³ volume of compressed air.

**Given:** 3.55 kg of air is isothermally compressed from state 1 to state 2 by a piston/cylinder.

- $m_1 := 3.55\text{kg}$
- $kPa := 1000\text{Pa}$
- $V_1 := 3\text{m}^3$
- $V_2 := 1.5\text{m}^3$
- $P_1 := 101.3\text{kPa}$
- $T_2 = T_1$ (isothermal)
- $r_p := 0.9\text{m}$ (piston radius)
- $P_{\text{atm}} := 101.3\text{kPa}$

**Find:**

a) $\rho_1$ in kg/m³, $T_1$ in K

b) $\rho_2$ in kg/m³, $P_2$ in kPa (abs)

c) $F_2$ (force applied to piston at state 2 in N)

**Schematic:**

[Diagram of the piston/cylinder system with states 1 and 2, and pressure and force annotations]
Assumptions:

1. No air leakage from piston/cylinder, therefore \( m_1 = m_2 \)
2. Air is an Ideal Gas
3. No friction between piston and cylinder
4. Piston is "weightless"

Properties:

\[
\begin{align*}
R_{\text{air}} &:= 286.9 \frac{\text{J}}{\text{kg} \cdot \text{K}} \quad \text{from Table 1.8 front cover} \\
k_{\text{air}} &:= 1.4
\end{align*}
\]

Solution:

a) density = mass/volume, \( \rho \):=

\[
\rho_1 := \frac{m_1}{V_1} \quad \rho_1 = 1.183 \frac{\text{kg}}{\text{m}^3}
\]

Ideal Gas, eqn 1.8, p. 14

\[
P := \rho \cdot R \cdot T \\text{therefore} \\ T := \frac{P}{\rho \cdot R}
\]

\[
T_1 := \frac{P_1}{\rho_1 \cdot R_{\text{air}}} \quad T_1 = 298.38 \text{ K}
\]

b) isothermal compression of an ideal gas

Eqn 1.14, p. 23

\[
\frac{P}{\rho} := \text{constant}
\]
\[ m_2 := m_1 \quad m_1 = 3.55 \text{ kg} \]

\[ \rho_2 := \frac{m_2}{V_2} \quad \rho_2 = 2.367 \frac{\text{kg}}{\text{m}^3} \]

\[ \frac{P_2}{\rho_2} := \frac{P_1}{\rho_1} \quad \text{therefore} \quad P_2 := P_1 \cdot \frac{\rho_2}{\rho_1} \]

\[ P_2 = 202.6 \text{ kPa} \]

c) \[ \sum F := 0 \quad 0 := P_2 \cdot A - P_{\text{atm}} \cdot A - F_2 \]

\[ A := \pi \cdot r_p^2 \quad \text{Area of piston} \]

\[ \text{therefore} \quad F_2 := (P_2 - P_{\text{atm}}) \cdot \pi \cdot r_p^2 \quad F_2 = 2.578 \times 10^5 \text{ N} \]