The Dot Product

The scalar product (a.k.a, “dot” product) is a way of multiplying two vectors together that results in a scalar. If $\vec{A}$ and $\vec{B}$ are two vectors, then their dot product is denoted as $\vec{A} \cdot \vec{B}$.

Dot Product – Geometric Representation:

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \phi$$

(Note: $\phi$ is the angle between $\vec{A}$ and $\vec{B}$ when lined up tail to tail.)

1. **Unit vectors.** Apply the above definition to all combinations of dot products between the unit vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$.

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<thead>
<tr>
<th>$\hat{i} \cdot \hat{i}$</th>
<th>$\hat{j} \cdot \hat{i}$</th>
<th>$\hat{k} \cdot \hat{i}$</th>
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2. **Dot Product – Algebraic Representation.** Let $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$ and $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$.

“FOIL” out the dot product $\vec{A} \cdot \vec{B}$. Show all intermediate steps. Use the above dot product rules for unit vectors to simplify the result. Thereby obtain an expression for the dot product in the algebraic representation (i.e., in terms of components).

$$\vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$= A_x B_x + A_y B_y + A_z B_z$$

$$= A \cdot B$$

$\Rightarrow \quad \vec{A} \cdot \vec{B}$ =

$$A \cdot B$$
3. \( \vec{A} \) has magnitude of 4 cm and is directed 30.0° counter-clockwise from the +x axis. \( \vec{B} \) has magnitude of 6 cm and is directed 45.0° counter-clockwise from the +x axis.

Calculate \( \vec{A} \cdot \vec{B} \).

4. Let \( \vec{F} = (2.00 \text{ N})\hat{i} + (3.00 \text{ N})\hat{j} - (4.00 \text{ N})\hat{k} \) and \( \vec{d} = -(2.00 \text{ m})\hat{j} + (5.00 \text{ m})\hat{k} \).

Calculate \( \vec{F} \cdot \vec{d} \).

5. Consider two vectors \( \vec{A} \) and \( \vec{B} \) in the \( xy \)-plane. \( \vec{A} \) has magnitude of 13.0 cm directed 15.0° counter-clockwise from the +x axis, and \( \vec{B} = (7.00 \text{ cm})\hat{i} - (9.00 \text{ cm})\hat{j} \).

(a) Express \( \vec{A} \) in terms of unit vectors.

(b) Find the magnitude and direction of \( \vec{B} \).

(c) Calculate \( \vec{A} \cdot \vec{B} \) using the geometric representation formula.

(d) Calculate \( \vec{A} \cdot \vec{B} \) using the algebraic representation formula.

6. Return to Problem 4. Determine the angle between \( \vec{F} \) and \( \vec{d} \).