Conservation of Energy III: Dynamics of Circular Motion

- If an object moves in a circle, the force causing it to move in a circle does no work.

  Reason: force is perpendicular to infinitesimal displacement at all points along the path, so \( \mathbf{F} \cdot d\mathbf{r} = 0 \).

**Example:** Tension in a pendulum does no work

\[
W_T = \int \mathbf{T} \cdot d\mathbf{r} = 0
\]

**Example:** Normal force in a loop-the-loop does no work

\[
W_N = \int \mathbf{N} \cdot d\mathbf{r} = 0
\]
Ex. Pendulum is released from rest from an angle of 42.0° with vertical. Pendulum bob has mass of 1.50 kg. What is the speed of the bob as it passes the lowest point (when pendulum passes through the vertical)? Ignore air resistance. Assume frictionless pivot and ideal string.

\[
\begin{align*}
\text{System} &= \text{bob + string + earth} \\
\text{between start and end:} \quad W_{\text{internal non-conservative}} &= W_{\text{tension}} = 0 \quad (\because \vec{T} \cdot d\vec{r} = 0) \\
W_{\text{external}} &= W_{\text{pivot}} = 0 \quad (\because \text{pivot doesn't move})
\end{align*}
\]

\[
\therefore \Delta E = 0
\]

At start: \( E_i = K_i + U_i \) 

At end: 
\[
E_f = K_f + U_f = \frac{1}{2} m v_f^2 + mg(-l(1 - \cos \theta)) = \frac{1}{2} m v_f^2 - mgl(1 - \cos \theta)
\]

\[
\Delta E = 0 \Rightarrow E_i = E_f \Rightarrow 0 = \frac{1}{2} m v_f^2 - mgl(1 - \cos \theta)
\]

\[
\Rightarrow v_f = \sqrt{2g \cdot l(1 - \cos \theta)} = \sqrt{2(9.8 \text{ m/s}^2)(2.00 \text{ m})(1 - \cos(42.0°))} \approx 3.3 \text{ m/s}
\]