Impulse, Momentum and Impulse-Momentum Theorem

- recall: definition of work \( W = \int F \cdot dx \)

\[ \begin{align*}
\text{dot product yields a scalar} \\
\text{scalar integral of force} \\
\text{with respect to space}
\end{align*} \]

Work-energy theorem \( W_{\text{tot}} = \Delta K \)

where we introduce a new \( \text{scalar} \) quantity \( K = \frac{1}{2} mv^2 \)

- in many ways, impulse is the complementary (or dual) quantity to work.

- definition of impulse: \( \vec{J} = \int \vec{F} \, dt \)

Remarks:

1. impulse is a vector rather than scalar
2. like work, impulse is an integral of force, but unlike work it is an integral with respect to time instead of space
3. no dot product involved because the result needs to be a vector (alternatively, impulse must be a vector, because time is not a vector, and so there is no way to get a scalar)
Impulse as area under a curve

(Note: $F_x$ vs. TIME, not $F_x$ vs. $x$)

Area under the curve gives work done on an object/system

Area under the curve gives impulse on an object/system
• impulse between two objects colliding:

\[ \text{no friction} \]

\[ A \quad M \quad B \]

\[ V_i \quad V_f \quad V_x \]

suppose \( M = 100 \text{m} \)
and \( |V_x| = 10 |V_i| \)

How does the impulse that \( A \) exerts on \( B \) during the collision compare to the impulse that \( B \) exerts on \( A \)?

Answer: by Newton's third law, equal but opposite

\[ F_x \]

impulse that \( A \) exerts on \( B \)

\[ F_{AmB,x} \]

\[ t \]

impulse that \( B \) exerts on \( A \)

\[ F_{BonA,x} \]

NOTE: this means that if we take \( A \) and \( B \) together, as a system, then this system experiences NO impulse during the collision (the impulses on the two individual parts cancel one another when they are summed).
• average force during a collision:

\[ \vec{F}_{av} = \frac{1}{\Delta t} \int_{t_1}^{t_2} \vec{F} \, dt = \frac{\vec{F}}{\Delta t} \]

NOTE: this is how a time average of a continuous variable is always defined.

for some quantity \( f(t) \), its time-average over an interval of time \( \Delta t \) is defined as

\[ f_{av} = \frac{1}{\Delta t} \int_{t_1}^{t_2} f(t) \, dt \]

We have actually been using this all along.

Recall:

\[ \vec{V}_{av} = \frac{\Delta \vec{V}}{\Delta t} = \frac{1}{\Delta t} (\vec{V}_2 - \vec{V}_1) = \frac{1}{\Delta t} \int_{t_1}^{t_2} \vec{V} \, dt \]

\[ \vec{A}_{av} = \frac{\Delta \vec{V}}{\Delta t} = \frac{1}{\Delta t} (\vec{V}_2 - \vec{V}_1) = \frac{1}{\Delta t} \int_{t_1}^{t_2} \vec{a}(t) \, dt \]

\[ \vec{P}_{av} = \frac{\vec{W}}{\Delta t} = \frac{1}{\Delta t} \int \vec{F} \cdot d\vec{r} = \frac{1}{\Delta t} \int_{t_1}^{t_2} \vec{F} \cdot d\vec{r} \, dt = \frac{1}{\Delta t} \int_{t_1}^{t_2} \vec{P} \, dt \]
impulse - momentum theorem:

\[ \int_{t_1}^{t_2} F_{\text{net}} \, dt = \int_{t_1}^{t_2} m \ddot{x} \, dt \]

\[ = m \int_{t_1}^{t_2} \ddot{x} \, dt \]

\[ = m \int_{t_1}^{t_2} \frac{dV}{dt} \, dt \]

\[ = m \int_{t_1}^{t_2} \frac{d\vec{V}}{dt} \, dt \]

\[ = m (\vec{V}_2 - \vec{V}_1) \]

\[ = \Delta \vec{p} \]

\[ \int_{t_1}^{t_2} F_{\text{net}} \, dt = \Delta \vec{p} \]

\[ \Rightarrow \]

IMPULSE - MOMENTUM THEOREM

\( \Delta \vec{p} \) says that the total impulse on an object/system is equal to the object/system's change in momentum.

\( \Rightarrow \) assume \( m \) = constant in time

\( \Rightarrow \) definition of acceleration

\( \Rightarrow \) chain rule

\( \Rightarrow \) define new quantity called (linear) momentum: \( \vec{p} = m \vec{V} \)
1. **Relationship between force and momentum:**

\[
\vec{F}_{\text{net}} = \frac{\vec{F}_{\text{tot}}}{\Delta t} = \frac{\Delta \vec{p}}{\Delta t}
\]

Impulse-Momentum Theorem

\[
\vec{F}_{\text{net}} = \lim_{\Delta t \to 0} \frac{\vec{F}_{\text{net}}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta \vec{p}}{\Delta t} = \frac{d\vec{p}}{dt}
\]

\[
\Rightarrow \quad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}
\]

2. **Internal vs. External forces:**

\[
\vec{F}_{\text{net}} = \vec{F}_{\text{net, internal}} + \vec{F}_{\text{net, external}}
\]

always equal to zero, by Newton's 3rd law

\[
\Rightarrow \quad \vec{F}_{\text{net}} = \vec{F}_{\text{net, external}}
\]

\[
\Rightarrow \quad \vec{F}_{\text{net, ext}} = \frac{d\vec{p}}{dt}
\]
Ex: Puck hitting a wall

A puck of mass 1.50 kg collides head on with a wall. The puck's speed before the collision is 3.00 m/s. The puck's speed after the collision is 2.98 m/s. Assume no friction between the puck and horizontal surface (ice?) that it slides on. Also assume air resistance is negligible.

(a) What is the impulse (magnitude and direction) that the wall exerts on the puck?

(b) If the collision lasts for 10.0 ms, what is the avg. force that the wall exerts on the puck?

\[
\begin{array}{c}
\text{top view before:} \\
& \downarrow \\
& x \\
& y \\
& O \\
& \vec{v}_i \\
\end{array}
\quad
\begin{array}{c}
\text{top view after:} \\
& \downarrow \\
& x \\
& y \\
& \vec{v}_f \quad \vec{k} = 0 \\
\end{array}
\]

(a) By the impulse-momentum theorem:

\[
\vec{J}_{\text{tot}} = \Delta \vec{P}_{\text{puck}}
\]

Since the net force acting on the puck during the collision is the force from the wall (normal force from surface is balanced by gravitational force from earth),

\[
\vec{J}_{\text{wall on puck}} = \vec{J}_{\text{puck}}
\]

In x-direction:

\[
\begin{align*}
\vec{J}_{\text{tot,x}} &= \vec{J}_{\text{wall on puck, x}} = \Delta P_{\text{x,puck}} = P_{\text{fx,puck}} - P_{\text{fx,puck}} \\
&= M_{\text{puck}}(V_{\text{fx,puck}} - V_{\text{x,puck}}) = (1.50 \text{ kg})(2.98 \text{ m/s} - 3.00 \text{ m/s}) \\
&= -8.97 \text{ kg m/s}
\end{align*}
\]
(b) \[ F_{av} = \frac{\overrightarrow{F}_{(wall \ on \ puck)}}{\Delta t} = \frac{\overrightarrow{F}_{(puck)}}{\Delta t} = \frac{\Delta \overrightarrow{p}_{(puck)}}{\Delta t} \]

in x-direction: \[ F_{av,x} = \frac{\overrightarrow{F}_{(wall \ on \ puck)}}{\Delta t} = \frac{\Delta \overrightarrow{p}_{x}(puck)}{\Delta t} = -\frac{8.97 \text{ kg m/s}}{10 \times 10^{-3} \text{ s}} = -897 \text{ N} \]

\[ F_{av} \] has magnitude of 897 N, and is directed in -x direction (away from the wall)

- It is crucial to recognize that impulse and momentum are vectors, not scalars. That means all equations that involve impulse and momentum, such as the impulse-momentum theorem, must be treated component-wise.

**Exercise:** Puck hitting a wall, revisited

Consider the same puck as in the previous problem (mass of 1.50 kg, sliding on frictionless horizontal surface, with initial speed of 3.00 m/s), but this time the puck strikes the wall at an angle of 15.0° relative to the wall. Assume the magnitude of the impulse in this case is 25% that of the previous example. Also assume that there is no friction between the wall and the puck.
(a) What is the direction of the impulse from the wall on the puck?

(b) Will the angle that the puck's velocity makes with the wall after the collision be greater than, equal to, or less than the initial angle (before the collision)?

(c) Find the final speed velocity (speed and direction) of the puck.

**Free-body diagram of puck during collision with wall:**
(at an arbitrary instant during the collision)

**Top view:**

**Front view:**

(Note: there can be no forces parallel to the wall. These would be friction forces, but there is no friction between wall and puck (by assumption).
so as before, \( F_{\text{net}} = N_{\text{wall on puck}} \).

Also as before, \( N_{\text{wall on puck}} \) is perpendicular to the wall.

so if we choose coordinates:

\[
\begin{array}{c}
\text{y} \\
\downarrow \\
\text{x}
\end{array}
\]

then

\[
\dot{J}_{t,x}^{(\text{puck})} = -N_{\text{wall on puck}} \cdot \Delta t = -0.25 \text{ (impulse from previous example)} = -2.24 \text{ kg} \cdot \text{m/s}
\]

and

\[
\dot{J}_{t,y}^{(\text{puck})} = 0 \text{ (no forces in y-direction)}
\]

\[
\Rightarrow \quad \dot{J}_{t}^{(\text{puck})} \text{ has magnitude } 2.24 \text{ kg} \cdot \text{m/s} \text{, and is directed in the } -x \text{ direction (away from the wall)}
\]

(b) by the impulse-momentum theorem:

\[
\Delta P_{x}^{(\text{puck})} = \dot{J}_{t,x}^{(\text{puck})} \neq 0
\]

\[
\Delta P_{y}^{(\text{puck})} = \dot{J}_{t,y}^{(\text{puck})} = 0 \Rightarrow y\text{-momentum remains constant (y-momentum is "conserved")}
\]
so final angle depends on final x-momentum in comparison to initial x-momentum:

\[
\text{if } |P_{fx}^{(puck)}| = |P_{ix}^{(puck)}|, \quad \phi = \Theta
\]

\[
\text{if } |P_{fx}^{(puck)}| < |P_{ix}^{(puck)}|, \quad \phi < \Theta
\]

\[
\text{if } |P_{fx}^{(puck)}| > |P_{ix}^{(puck)}|, \quad \phi > \Theta
\]

by impulse-momentum and result from part (a):

\[
\Delta P_x^{(puck)} = J_{tot,x}^{(puck)} = -2.24 \text{ kg m/s}
\]

\[
\Rightarrow P_{fx}^{(puck)} - P_{ix}^{(puck)} = -2.24 \text{ kg m/s}
\]

\[
\Rightarrow P_{fx}^{(puck)} = -2.24 \text{ kg m/s} + P_{ix}^{(puck)}
\]

\[
= -2.24 \text{ kg m/s} + (1.50 \text{ kg})(3.00 \text{ m/s}) \sin(15.0^\circ) = -1.08 \text{ kg m/s}
\]
\[ \Rightarrow |P_{fx}^{(puck)}| < |P_{ix}^{(puck)}| \]

\[ \therefore \phi < \theta \quad \text{(outgoing angle with wall will be less than the ingoing angle with wall)} \]

(c) from above:

\[ P_{fx}^{(puck)} = -1.08 \text{ kg m/s} \]

\[ P_{fy}^{(puck)} = P_{iy}^{(puck)} = -m_{puck} V_i^{(puck)} \cos \theta \]

\[ = -(1.50 \text{ kg})(3.00 \text{ m/s}) \cos(15.0^\circ) \]

\[ = -4.35 \text{ kg m/s} \]

\[ \left( \frac{P_{fy}^{(puck)}}{P_{fx}^{(puck)}} = m_{puck} \right), \text{ so direction of } P_f^{(puck)} \text{ is the same as direction of } V_f^{(puck)} \]

\[ \phi = \tan^{-1} \left( \frac{|P_{fx}^{(puck)}|}{|P_{fy}^{(puck)}|} \right) = \tan^{-1} \left( \frac{1.08}{4.35} \right) = 13.9^\circ \]

\[ V_f^{(puck)} = \frac{P_f^{(puck)}}{m_{puck}} = \frac{1}{m_{puck}} \sqrt{\left(\frac{P_{fx}^{(puck)}}{P_{fy}^{(puck)}}\right)^2 + \left(\frac{P_{fy}^{(puck)}}{P_{fx}^{(puck)}}\right)^2} \]

\[ = \frac{1}{(1.50 \text{ kg})} \sqrt{(-1.08 \text{ kg m/s})^2 + (-4.35 \text{ kg m/s})^2} \]

\[ = 2.99 \text{ m/s} \]

\[ \Rightarrow V_f^{(puck)} \text{ has magnitude of 2.99 m/s, directed } 13.9^\circ \text{ clockwise from } -y \text{ axis.} \]
- momentum of a system:

\[
\vec{p}_{\text{(system)}} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots = \sum_{i=1}^{N} \vec{p}_i
\]  
(assuming N objects included in the system)

- conservation of momentum:

If \( \vec{F}_{\text{tot}} = 0 \), then by impulse-momentum theorem

\[
\Delta \vec{p}_{\text{(system)}} = 0, \quad \text{i.e., } \vec{p}_{\text{(system)}} = \text{constant.}
\]

Equivalently, if \( \vec{F}_{\text{net ext}} = 0 \), then by \( \frac{d\vec{p}}{dt} = \vec{F}_{\text{net ext}} \),

we have \( \frac{d\vec{p}_{\text{(system)}}}{dt} = 0 \), \text{i.e., } \vec{p}_{\text{(system)}} = \text{constant.}

\[
\begin{align*}
\vec{F}_{\text{tot}} = 0 & \iff \Delta \vec{p}_{\text{(system)}} = 0 & \iff \vec{p}_{f} = \vec{p}_{i} \\
\vec{F}_{\text{net ext}} = 0 & \iff \frac{d\vec{p}_{\text{(system)}}}{dt} = 0 & \iff \vec{p}_{f} = \vec{p}_{i}
\end{align*}
\]
Ex. Collision between two blocks on a frictionless surface

before collision: \[ \begin{array}{c}
\text{A} \rightarrow \\
\text{B} \leftarrow
\end{array} \]

after collision: \[ \begin{array}{c}
\text{A} \rightarrow \\
\text{B} \rightarrow
\end{array} \]

Is momentum of A by itself conserved throughout the collision?
To see, consider free-body diagram of A at some instant during the collision:

\[ F_{\text{B on A}} \]
\[ N_A \]
\[ m_A g \]
\[ \vec{F}_{\text{ext}}(A) = 0 \] during the collision?
Answer: No! Therefore \( \Delta \vec{p}(A) \neq 0 \), i.e., A's momentum is not constant, i.e., not conserved, i.e., is changed by the collision.

Is momentum of B by itself conserved throughout the collision?
FBD of B:

\[ \begin{array}{c}
\text{B} \rightarrow \\
\text{F on B} \leftarrow
\end{array} \]
\[ m_B g \]
\[ \text{net external force on B} \Rightarrow \Delta \vec{p}(B) \neq 0. \]

\[ \therefore \text{B's momentum is not conserved during collision.} \]

However, the momentum of the A+B system \( \neq \) conserved during the collision. (Why?)
• solving problems using impulse-momentum methods:

1. identify what you mean by "the system".
   (This is a choice, although oftentimes certain choices are
   more tractable than others... see flow chart below)

2. identify instants of interest
   (NOTE: collisions take time, so "the collision" is not an instant.
   Instead identify "the instant at the beginning of the collision" and "the
   instant at the end of the collision" as two different instants.)

3. Ask:
   Is \( \vec{F}_{\text{net}} = 0 \) at all times between
   the instants of interest?
   
   \begin{align*}
   &\text{Yes} \quad \downarrow \quad \text{No} \\
   &\text{momentum of} \quad \quad \quad \quad \text{momentum of the} \\
   \text{the system is} \quad \text{system is not conserved}
   \text{conserved between instants of interest} \\
   \text{i.e., } \Delta \vec{p} = 0 \\
   \text{i.e., } \vec{p}_{\text{final}} = \vec{p}_{\text{initial}} \\
   \end{align*}

   Are you given enough information
   to calculate \( \vec{F}_{\text{tot}} \) directly (i.e.,
   using definition of \( \vec{F} \), not using
   impulse-momentum theorem)?

   \begin{align*}
   &\text{Yes} \quad \downarrow \quad \text{No} \\
   &\text{ok, do it! } \\
   \text{Then use } \vec{p}_f = \vec{p}_i + \vec{F}_{\text{tot}} \\
   \end{align*}

   Then you cannot proceed.
   Either go back and re-define your
   system, or try solving using different
   methods (e.g., energy, forces, kinematics)
4. Write symbolic expressions for the components of $\mathbf{p}_i^{(\text{system})}$

\[
\begin{align*}
\mathbf{p}_{ix}^{(\text{system})} &= \mathbf{p}_{ix}^{(\text{object 1})} + \mathbf{p}_{ix}^{(\text{object 2})} + \mathbf{p}_{ix}^{(\text{object 3})} + \ldots \\
\mathbf{p}_{iy}^{(\text{system})} &= \mathbf{p}_{iy}^{(\text{object 1})} + \mathbf{p}_{iy}^{(\text{object 2})} + \mathbf{p}_{iy}^{(\text{object 3})} + \ldots \\
\mathbf{p}_{iz}^{(\text{system})} &= \mathbf{p}_{iz}^{(\text{object 1})} + \mathbf{p}_{iz}^{(\text{object 2})} + \mathbf{p}_{iz}^{(\text{object 3})} + \ldots
\end{align*}
\]

5. Write symbolic expressions for the components of $\mathbf{p}_f^{(\text{system})}$

\[
\begin{align*}
\mathbf{p}_{fx}^{(\text{system})} &= \mathbf{p}_{fx}^{(\text{object 1})} + \mathbf{p}_{fx}^{(\text{object 2})} + \ldots \\
\mathbf{p}_{fy}^{(\text{system})} &= \mathbf{p}_{fy}^{(\text{object 1})} + \mathbf{p}_{fy}^{(\text{object 2})} + \ldots \\
\mathbf{p}_{fz}^{(\text{system})} &= \mathbf{p}_{fz}^{(\text{object 1})} + \mathbf{p}_{fz}^{(\text{object 2})} + \ldots
\end{align*}
\]

6. Write symbolic expressions for the components of $\mathbf{f}_{\text{ext}}^{(\text{system})}$

\[
\begin{align*}
\mathbf{f}_{\text{ext},x}^{(\text{system})} &= \ldots \\
\mathbf{f}_{\text{ext},y}^{(\text{system})} &= \ldots \\
\mathbf{f}_{\text{ext},z}^{(\text{system})} &= \ldots
\end{align*}
\]
7. Putting it all together, write symbolic expressions for component-wise impulse-momentum equations:

\[ P_{\text{fx}}^{(\text{system})} = p_{\text{fx}} + F_{\text{ext},x}^{(\text{system})} \]
\[ P_{\text{fy}}^{(\text{system})} = p_{\text{fy}} + F_{\text{ext},y}^{(\text{system})} \]
\[ P_{\text{fz}}^{(\text{system})} = p_{\text{fz}} + F_{\text{ext},z}^{(\text{system})} \]

8. Ask:

Do you have as many equations as unknowns?

\[
\begin{array}{c}
\text{Yes} \\
\text{great!} \\
\text{solve!} \\
\end{array}
\quad
\begin{array}{c}
\text{No} \\
\text{Then there is more} \\
\text{information that you have} \\
\text{not utilized (or the problem} \\
\text{is unsolvable).} \\
\end{array}
\]


10. Check reasonableness of your answer.

(limiting cases, etc.)

Think about whether you've made a mistake, and whether there is more physics in the problem than simply momentum and impulse (think work-energy, Newton's laws, kinematics, or other considerations). Re-read the problem carefully to see if you missed something (or misinterpreted something).
Ex. One dimensional collision

Cart of mass 11.0 kg moving to the right at 3.00 m/s and a block of mass 5.0 kg moving to the left at 12.0 m/s collide while moving along a frictionless horizontal surface. After the collision, the block moves to the right at 4.00 m/s.

(a) What is the velocity (magnitude and direction) of the cart after the collision?

(b) If the collision lasts for 20.0 ms, find the avg. force on the cart during the collision.

(c) Find the avg. force on the block during the collision.

(d) Is kinetic energy conserved during the collision?

Choose system: I will take the system to include cart and block

FBD's during collision:

Cart
\[ \vec{N}_c \]  
\[ \vec{F}_{f_{\text{c}}} \]  
\[ \vec{F}_{\text{c}} \]  
\[ m_c \vec{g} \]

Block
\[ \vec{N}_b \]  
\[ \vec{F}_{c_{\text{on b}}} \]  
\[ \vec{F}_{\text{b}} \]  
\[ m_b \vec{g} \]
System

\[ \vec{N}_c \rightarrow \vec{N}_b \]

\[ \text{m}_c \rightarrow \text{m}_b \]

\[ F_{\text{comb}} \text{ and } F_{\text{b on c}} \]

are a third-law pair, and so cancel out when considering the cart + block system.

\[ F_{\text{net}} = 0 \]

\[ \Delta \vec{P} = 0 \]

\[ (\Delta \vec{P} = \vec{J}_{\text{tot}}), \text{ or } \frac{d\vec{P}}{dt} = \vec{F}_{\text{net}} \]

\[ \Rightarrow \vec{J}_{\text{tot}} = 0 \Rightarrow \Delta \vec{P} = 0 \Rightarrow \vec{P} = \text{constant} \]

Initial x-momentum:

\[ P_{ix}^{(\text{system})} = M_c V_{ci} - m_b V_{bi}; \quad = P_{ix}^{(\text{cart})} + P_{ix}^{(\text{block})} \]

Final x-momentum:

\[ P_{fx}^{(\text{system})} = P_{fx}^{(\text{cart})} + P_{fx}^{(\text{block})} = m_c V_{cf,x} + m_b V_{bf} \]

by conservation of momentum in the x-direction:

\[ \Delta P_{ix}^{(\text{system})} = 0 \Rightarrow P_{fx}^{(\text{system})} = P_{ix} \]

\[ \Rightarrow M_c V_{cf,x} + M_b V_{bf} = M_c V_{ci} - M_b V_{bi} \]
\[ \Rightarrow m_c V_{c_f, x} = m_c V_{c_i} - m_b V_{b_i} - m_b V_{b_f} \]
\[ = m_c V_{c_i} - m_b (V_{b_i} + V_{b_f}) \]
\[ \Rightarrow V_{c_f, x} = V_{c_i} - \frac{m_b}{m_c} (V_{b_i} + V_{b_f}) \]
\[ = (3.00 \text{ m/s}) - \frac{(5.00 \text{ kg})}{(11.0 \text{ kg})} (12.0 \text{ m/s} + 4.00 \text{ m/s}) \]
\[ = -4.27 \text{ m/s} \]

\[ \Rightarrow \overrightarrow{V_{c_f}} \text{ has magnitude of } 4.27 \text{ m/s}, \text{ in } -x \text{ direction.} \]

(b) \[ \overrightarrow{F_{avg}} = \frac{\overrightarrow{J}_{tot}}{\Delta t} = \frac{\Delta \overrightarrow{P}_{(\text{cart})}}{\Delta t} \]
\[ \Rightarrow F_{av, x} = \frac{\Delta P_{x}}{\Delta t} = \frac{P_{f_x} - P_{i_x}}{\Delta t} = \frac{m_c (V_{c_f, x} - V_{c_i, x})}{\Delta t} \]
\[ = \frac{(11.0 \text{ kg})}{(0.020 \text{ s})} (-4.27 \text{ m/s} - 3.00 \text{ m/s}) \]
\[ = -4000 \text{ N} \]

\[ \Rightarrow \overrightarrow{F_{av}} \text{ has magnitude of } 4000 \text{ N}, \text{ directed in the } -x \text{ direction.} \]

(c) by Newton's third law, \[ \overrightarrow{F_{av}} = -\overrightarrow{F_{av}} \quad (4000 \text{ N in the } +x \text{ direction}) \]
on one can verify this with a direct calculation:
\[ F_{av, x} = \frac{\Delta P_{x}}{\Delta t} = \frac{P_{f_x} - P_{i_x}}{\Delta t} = \frac{m_b}{\Delta t} (V_{b_f, x} - V_{b_i, x}) \]
\[ F_{av,x} = \left( \frac{5.00 \, \text{kg}}{0.020 \, \text{kg}} \right) \left( 4.00 \, \text{m/s} - (-12.0 \, \text{m/s}) \right) \]
\[ = +4000 \, \text{N} \]

(d) Check if \( \Delta K = 0 \).

\[ K_i^{(\text{system})} = K_i^{(\text{cart})} + K_i^{(\text{block})} \]
\[ = \frac{1}{2} m_e V_e^2 + \frac{1}{2} m_b V_{bi}^2 \]
\[ = \frac{1}{2} (11.0 \, \text{kg})(3.00 \, \text{m/s})^2 + \frac{1}{2} (5.00 \, \text{kg})(12.0 \, \text{m/s})^2 \]
\[ = 409.5 \, \text{J} \]

\[ K_f^{(\text{system})} = K_f^{(\text{cart})} + K_f^{(\text{block})} \]
\[ = \frac{1}{2} m_e V_{ef}^2 + \frac{1}{2} m_b V_{bf}^2 \]
\[ = \frac{1}{2} (11.0 \, \text{kg})(4.27 \, \text{m/s})^2 + \frac{1}{2} (5.00 \, \text{kg})(4.00 \, \text{m/s})^2 \]
\[ = 140.3 \, \text{J} \]

\[ \Delta K = K_f^{(\text{system})} - K_i^{(\text{system})} = 140.3 \, \text{J} - 409.5 \, \text{J} \]
\[ = -269.2 \, \text{J} \]

\[ \therefore \text{kinetic energy has been lost during the collision.} \]

(we will see that this kind of collision is called inelastic)