We saw that many quantities from linear mechanics had rotational analogs, and that when the relevant substitutions are made, the correct kinematic equations are obtained.

We will now see that the same thing holds more generally for dynamics.

In particular, Newton's second law has a rotational analog.

Recall:
\[ \vec{F}_{\text{net}} = m \vec{a} \]

\( \vec{F}_{\text{net}} \) = net force
\( m \) = quantity of inertia (inertial mass)
\( \vec{a} \) = linear acceleration

Now:
\[ \vec{T}_{\text{net}} = I \vec{\alpha} \]

\( \vec{T}_{\text{net}} \) = net rotational force ("torque")
\( I \) = rotational inertia ("moment of inertia")
\( \vec{\alpha} \) = angular acceleration
\[ \begin{array}{|l|l|l|l|}
\hline
\text{linear quantity} & \text{rotational quantity} & \text{units} & \text{scalar or vector} \\
\hline
\text{force } (\vec{F}) & \text{torque } (\vec{\tau}) & \text{Nm} & \text{vector} \\
\text{mass } (m) & \text{moment of inertia } (I) & \text{kg}\cdot\text{m}^2 & \text{scalar} \\
\hline
\end{array} \]

Just as acceleration of an object is proportional to, and in the same direction as, the net force applied to the object, in rotational mechanics the angular acceleration of an object (about an axis) is proportional to, and in the same direction as, the net torque applied to the object (about the same axis).

\[ \vec{F}_{\text{net}} \propto \vec{a} \]

\[ \vec{\tau}_{\text{net}} \propto \vec{\alpha} \]

And just as inertial mass can be thought of as the proportionality constant between \( \vec{F}_{\text{net}} \) and \( \vec{a} \), moment of inertia can be thought of as a proportionality constant between \( \vec{\tau}_{\text{net}} \) and \( \vec{\alpha} \):

\[ m = \frac{\vec{F}_{\text{net}}}{\vec{a}} \quad , \quad I = \frac{\vec{\tau}_{\text{net}}}{\vec{\alpha}} \]
Torque and Cross Product

torque - is a twisting force about a point

depends on:
1. applied force, magnitude
2. length of lever arm
3. applied force, direction

- different applied force magnitudes
- same applied force direction
- same lever arm length

- different lever arm lengths
- same applied force
  (magnitude and direction)

- different applied force directions
- same applied force magnitude
- same lever arm length
torque of a force, \( \vec{F} \), about a point with lever arm \( \vec{r} \):

\[
\vec{r} \times \vec{F}
\]

lever arm points from the point around which the torque is being calculated to the point where the force is applied.

\[
|\vec{r}| = |F||F'| \sin \theta
\]

where \( \theta \) is the angle between \( \vec{F} \) and \( \vec{F}' \) when they are aligned tail to tail.

Examples:

No:

\[
\begin{align*}
\vec{F} & \quad \text{(not aligned tail to tail)} \\
mg & \quad \text{(not aligned tail to tail)}
\end{align*}
\]

Yes:

\[
\begin{align*}
\vec{F} & \quad \text{(vectors aligned tail to tail)} \\
mg & \quad \text{(vectors aligned tail to tail)}
\end{align*}
\]

\[
\theta = 120^\circ \neq 30^\circ \neq 60^\circ
\]

\[
\Rightarrow \text{torque exerted about point } O \text{ due to } mg: \quad |\vec{r}| = mgL \sin \theta
\]