Today's topic: frequency response

Chapter 4

Small-signal analysis applies when transistors can be adequately characterized by their operating points and small linear changes about the points.

The use of this technique has led to application of frequency-domain techniques to the analysis of the linear equivalent circuits derived from small-signal models.

The transfer function of analog circuits to be discussed can be written in rational form with real-valued coefficients, that is as a ratio of polynomials in Laplace Transform variable s,

$$H(s) = \frac{a_0 + a_1 s + \dots + a_m s^m}{1 + b_1 s + \dots + b_n s^n}$$

$$H(s) = K\frac{(s+z_1)(s+z_2)\dots(s+z_m)}{(s+\omega_1)(s+\omega_2)\dots(s+\omega_n)} = a_0\frac{\left(1+\frac{s}{z_1}\right)\left(1+\frac{s}{z_2}\right)\dots\left(1+\frac{s}{z_m}\right)}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)\dots\left(1+\frac{s}{\omega_n}\right)}$$

$$H(s) = \frac{a_0 + a_1 s + \dots + a_m s^m}{1 + b_1 s + \dots + b_n s^n} \qquad H(s) = K\frac{(s + z_1)(s + z_2)\dots(s + z_m)}{(s + \omega_1)(s + \omega_2)\dots(s + \omega_n)}$$

$$= a_0 \frac{\left(1 + \frac{s}{z_1}\right)\left(1 + \frac{s}{z_2}\right)\dots\left(1 + \frac{s}{z_m}\right)}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)\dots\left(1 + \frac{s}{\omega_n}\right)}$$

The transfer function evaluated on the imaginary axis, $H(j\omega_{in})$, is its *frequency response* which may written in terms of its magnitude response $|H(j\omega_{in})|$ and phase response $\phi = \angle H(j\omega_{in})$.

$$H(j\omega_{in}) = |H(j\omega_{in})|e^{j\phi} \tag{4.11}$$

zeros are $-z_1, -z_2\dots$ and the poles are $-\omega_1, -\omega_2\dots$. It is also common to refer to the *frequencies* of the zeros and poles which are always positive quantities, $|z_1|, |z_2|\dots$ and $|\omega_1|, |\omega_2|\dots$, since these are physical frequencies of significance in describing the system's input-output behavior.

Key Point: *The transfer functions in analog circuits throughout this text:*
a) are rational with $m \le n$
b) have real-valued coefficients, a_i *and* b_i
c) have poles and zeros that are either real or appear in complex-conjugate pairs
Furthermore, if the system is stable,
d) all denominator coefficients $b_i > 0$
e) the real part of all poles will be negative

4.1.2 First order circuits

A first-order transfer function has a first order denominator, $n = 1$. For example,

$$H(s) = \frac{A_0}{1 + \frac{s}{\omega_0}}$$

It is a first-order low-pass transfer function.
It arises naturally when a resistance and capacitance are combined.
It is often used as a simple model of more complex circuits, such as OpAmp.

$$\phi = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

Note that

$$|H(\omega)| = \frac{A_0}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

Step response of first order circuits

Another common means of characterizing linear circuits is to excite the with step inputs (such a square waveform).

Consider a step input $x_{in}(t) = A_{in}u(t)$ where $u(t)$, the step-input function, is defined as

$$u(t) = \begin{cases} 0, & t \le 0 \\ 1, & t > 0 \end{cases}$$

The step function $u(t)$ has a Laplace transform given by

$$U(s) = \frac{1}{s}$$

$$X_{out}(s) = A_{in}\frac{H(s)}{s} \quad X_{out}(s) = \frac{A_{in}}{s}\frac{A_0}{1 + \frac{s}{\omega_0}} = A_{in}A_0\left[\frac{1}{s} - \frac{1}{s + \omega_0}\right]$$

$$x_{out}(t) = u(t)A_{in}A_0[1 - e^{-t/\tau}] \quad \text{where } \tau = 1/\omega_0.$$

$$A(s) = \frac{A_0}{1 + \frac{s}{\omega_{-3dB}}}$$

Key Point: *For a first-order lowpass transfer function with dc gain* $A_0 >> 1$, *the unity gain frequency is* $\omega_{ta} \approx A_0 \omega_{-3dB}$ *and* $\angle A(\omega_{ta}) \approx 90°$.

4.1.3 second order low-pass H(s) with real poles

$$H(s) = \frac{K}{(1+s\tau_1)(1+s\tau_2)} = \frac{K}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)} = \frac{K\omega_{p1}\omega_{p2}}{(s+\omega_{p1})(s+\omega_{p2})}$$

The coefficients τ_1, τ_2 or ω_{p1}, ω_{p2} are either real and positive, or occur in complex-conjugate pairs.

$$H(s) = \frac{K\omega_{p1}\omega_{p2}}{\omega_{p1}\omega_{p2} + s(\omega_{p1}+\omega_{p2}) + s^2} = \frac{K\omega_0^2}{\omega_0^2 + s\frac{\omega_0}{Q} + s^2}$$

Here, w0 is the resonant or pole frequency and Q the quality factor, K the DC gain of H(s).

Equating yields
$$\begin{cases} \omega_0^2 = \omega_{p1}\omega_{p2} \\ \frac{\omega_0}{Q} = \omega_{p1} + \omega_{p2} \end{cases}$$

$$\omega_{p1}, \omega_{p2} = \frac{\omega_0}{2Q}\left(1 \pm \sqrt{1-4Q^2}\right)$$

$$|H(\omega)| = \frac{K}{\sqrt{1+\left(\frac{\omega}{\omega_{p1}}\right)^2}\sqrt{1+\left(\frac{\omega}{\omega_{p2}}\right)^2}}$$

$$\phi = -\tan^{-1}\left(\frac{\omega}{\omega_{p1}}\right) - \tan\left(\frac{\omega}{\omega_{p2}}\right)$$

w_{p1}, w_{p2} are widely-spaced real poles

For the special case of real poles $\omega_{p1} \ll \omega_{p2}$, we have $Q \ll 1$,

$$\sqrt{1-4Q^2} \cong 1-2Q^2$$

$$\omega_{p1} \cong \omega_0 Q$$

$$\omega_{p2} \cong \frac{\omega_0}{Q}$$

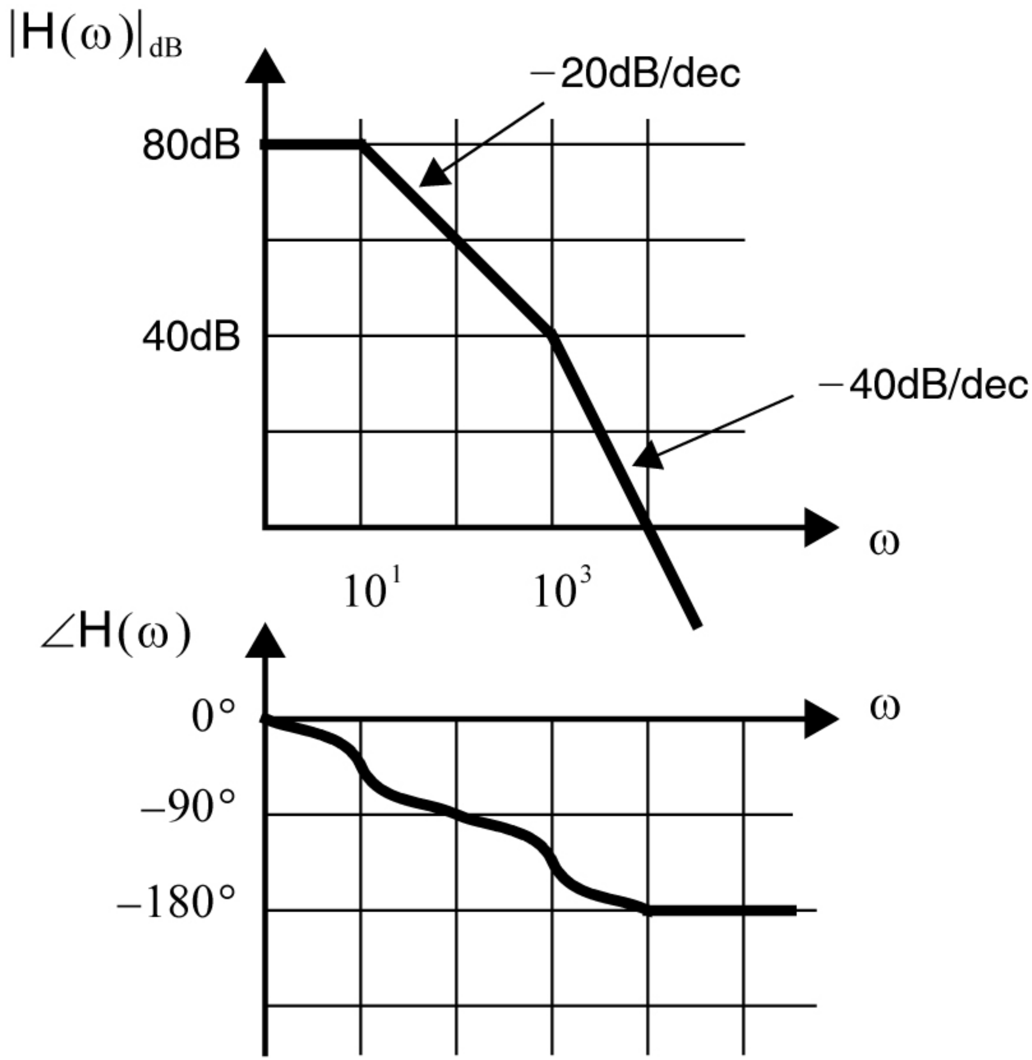
$$H(s) = K\left[\frac{\omega_{p2}}{\omega_{p2}-\omega_{p1}}\frac{\omega_{p1}}{s+\omega_{p1}} - \frac{\omega_{p1}}{\omega_{p2}-\omega_{p1}}\frac{\omega_{p2}}{s+\omega_{p2}}\right]$$

$$H(s) = K\left[\frac{\tau_1}{\tau_1-\tau_2}\frac{1}{1+s\tau_1} - \frac{\tau_2}{\tau_1-\tau_2}\frac{1}{1+s\tau_2}\right] \qquad \tau_1 = 1/\omega_{p1} \quad \text{and} \quad \tau_2 = 1/\omega_{p2}$$

$$x_{out}(t) = A_{in}\frac{K\omega_{p2}}{\omega_{p2}-\omega_{p1}}[1-e^{-t\omega_{p1}}] - A_{in}\frac{K\omega_{p1}}{\omega_{p2}-\omega_{p1}}[1-e^{-t\omega_{p2}}]$$

The step response consists of two first-order terms, and when $w_{p1} \ll w_{p2}$, the second settles fast and for $t \gg 1/w_{p2}$, the first term dominates.

4.1.4 Bode plot



Chapter 4 Figure 04

4.1.5 Second-order low-pass H(s) with complex poles

$$H(s) = \frac{K\omega_{p1}\omega_{p2}}{\omega_{p1}\omega_{p2} + s(\omega_{p1} + \omega_{p2}) + s^2} = \frac{K\omega_0^2}{\omega_0^2 + s\frac{\omega_0}{Q} + s^2}$$

$$\omega_{p1}, \omega_{p2} = \frac{\omega_0}{2Q}(1 \pm \sqrt{1 - 4Q^2})$$

when $Q > 1/2$ $\quad \omega_{p1}, \omega_{p2} = \frac{\omega_0}{2Q}[1 \pm j\sqrt{4Q^2 - 1}] \equiv \omega_r \pm j\omega_q$

$$\omega_r = \frac{\omega_0}{2Q}$$

$$\omega_q = \frac{\omega_0}{2Q}\sqrt{4Q^2 - 1}$$

$$|\omega_{p1}| = |\omega_{p2}| = \frac{\omega_0}{2Q}[1 + 4Q^2 - 1]^{1/2} = \omega_0$$

Recall $\quad x_{out}(t) = A_{in}\frac{K\omega_{p2}}{\omega_{p2} - \omega_{p1}}[1 - e^{-t\omega_{p1}}] - A_{in}\frac{K\omega_{p1}}{\omega_{p2} - \omega_{p1}}[1 - e^{-t\omega_{p2}}]$

Subt. In $\quad x_{out}(t) = \frac{A_{in}K}{-2j\omega_q}[(\omega_r - j\omega_q)(1 - e^{-\omega_r t}e^{-j\omega_q t}) - (\omega_r + j\omega_q)e^{-\omega_r t}e^{j\omega_q t}]$

$$= \frac{A_{in}K}{2j\omega_q}(j\omega_q[2 - e^{-\omega_r t}(e^{j\omega_q t} + e^{-j\omega_q t})] - \omega_r[e^{-\omega_r t}(e^{j\omega_q t} - e^{-j\omega_q t})])$$

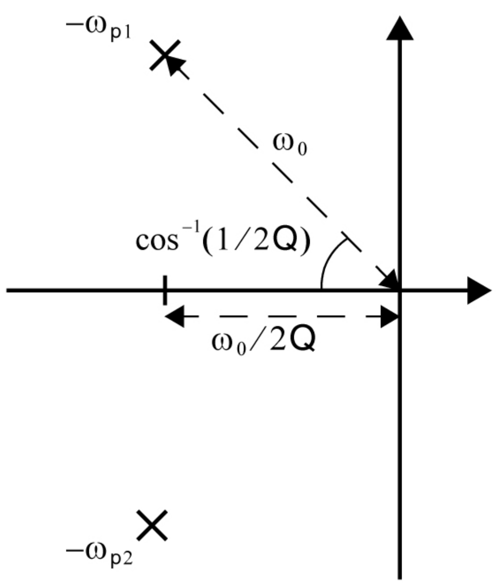
$$= A_{in}K\left[1 - e^{-\omega_r t}\cos(\omega_q t) - \frac{\omega_r}{\omega_q}e^{-\omega_r t}\sin(\omega_q t)\right]$$

$$= A_{in}K[1 - A_s e^{-\omega_r t}\cos(\omega_q t + \theta)]$$

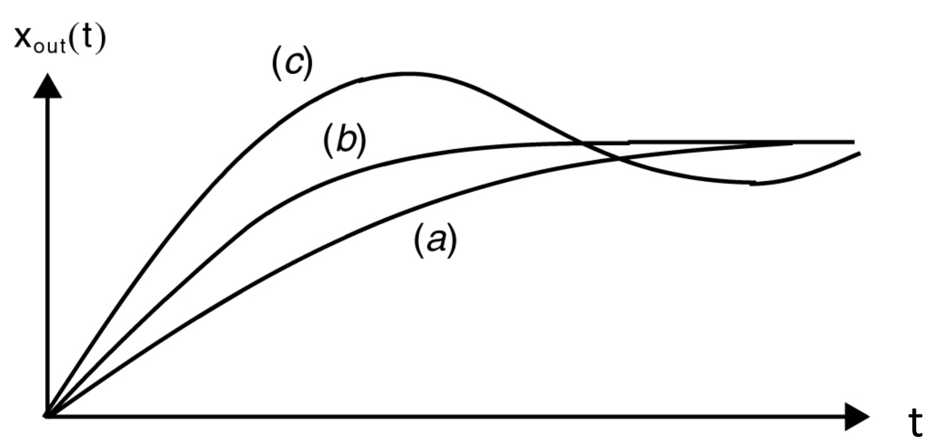
where $A_s = \sqrt{2 - 4Q^2}$ and $\theta = \tan^{-1}\sqrt{4Q^2 - 1}$.

4.1.5 Second-order low-pass H(s) with complex poles

1. The step response in this case has sinusoidal term whose envelope exponentially decays with a time constant equal to the inverse of real parts of poles, $1/w_r = 2Q/w_0$.
2. A system with high Q factor will have oscillation and ringing for some time. The oscillation frequency is determined by the imaginary parts of the poles.
3. In summary, when Q<0.5, the poles are real-valued and there is no overshoot. The borderline case Q=0.5 is called maximally-damped response. When Q>0,5, there are overshoot and ringing.



Chapter 4 Figure 09



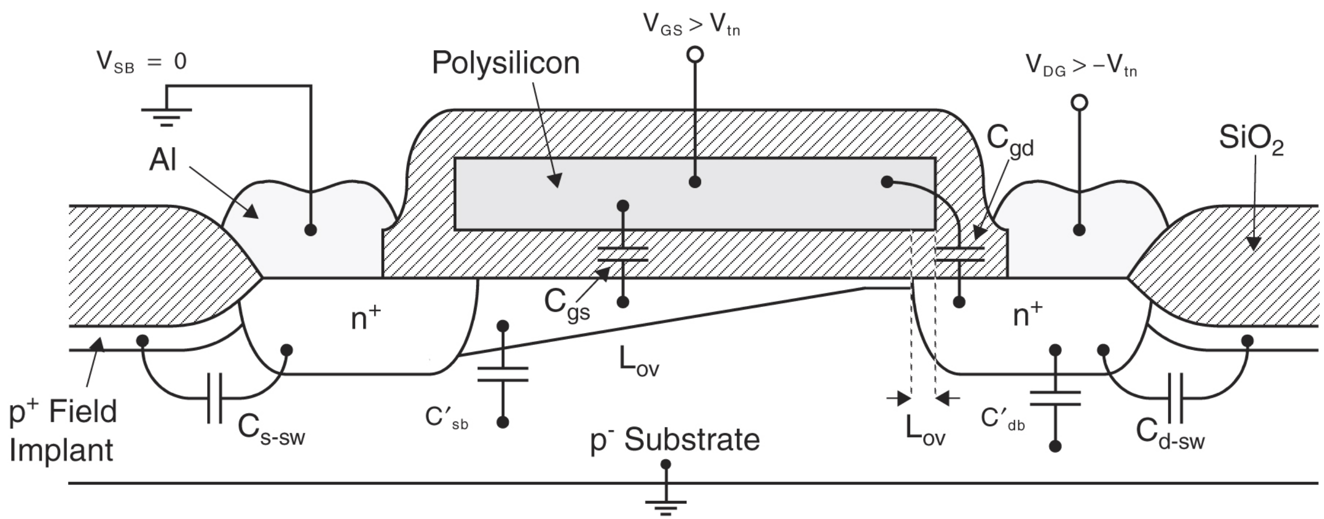
Chapter 4 Figure 10

4.2. Frequency response of elementary circuits

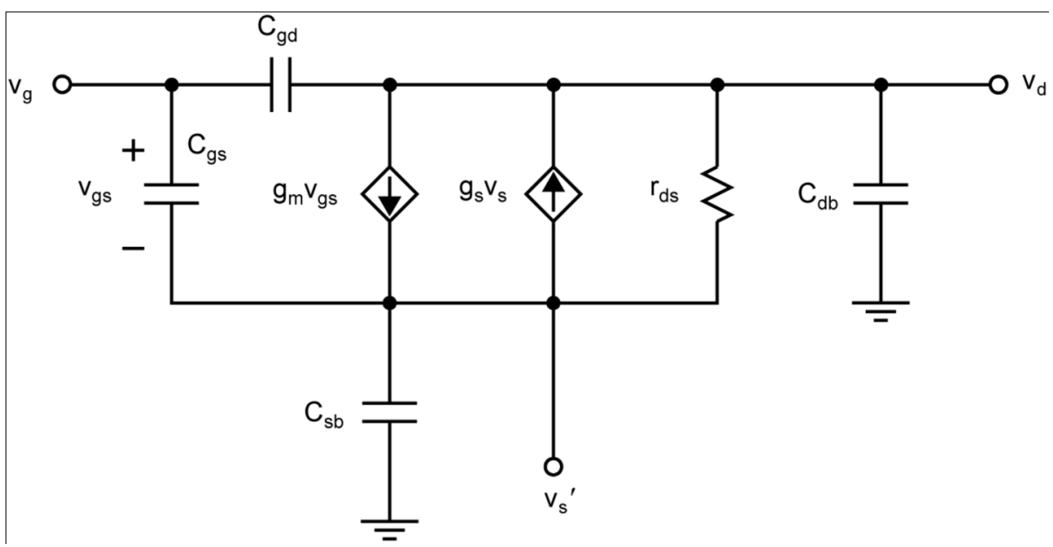
Small-signal analysis is implicitly assumed as only linear circuits can have well-defined frequency response.

The procedure for small-signal analysis remains the same as that in Chapter 3 for single-stage amplifiers, however parasitic capacitance are now included.

4.2.1 High frequency small-signal model

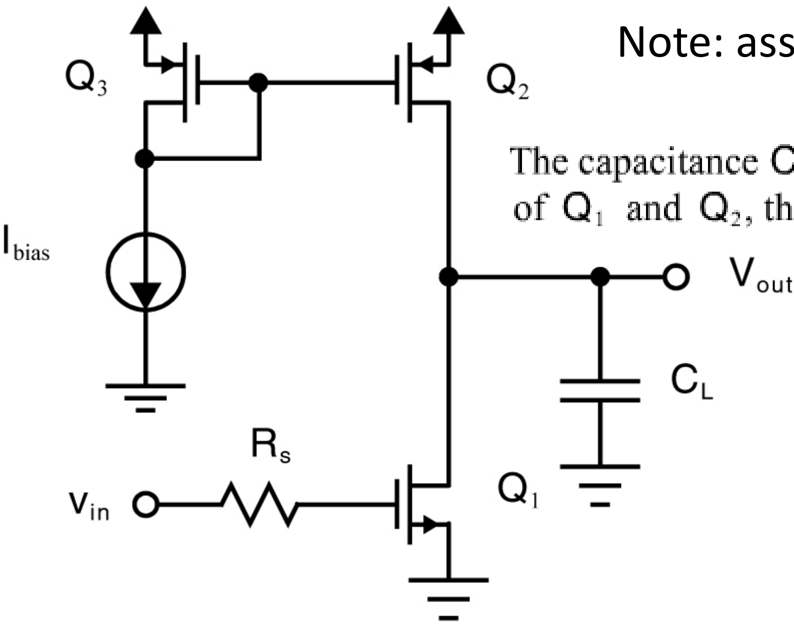


Chapter 4 Figure 11



Chapter 4 Figure 12

4.2.2 Common-source amplfier



Chapter 4 Figure 13

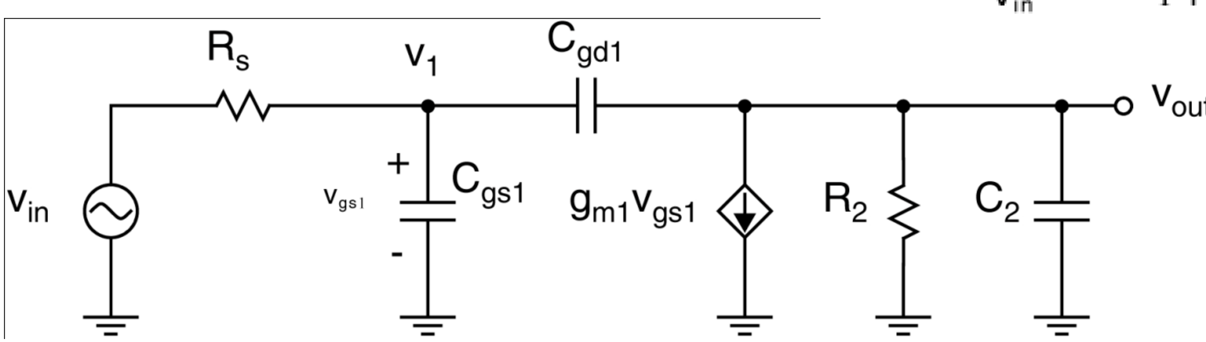
Note: assumed that Q1, Q2 are in active mode.

The capacitance C_2 is made up of the parallel connection of the drain-to-bulk capacitances of Q_1 and Q_2, the gate-drain capacitance of Q_2, and the load capacitance C_L.

$$C_2 = C_{db1} + C_{db2} + C_{gd2} + C_L$$

$$R_2 = r_{ds1} \| r_{ds2}$$

$$A(s) = \frac{V_{out}}{V_{in}} = \frac{-g_{m1}R_2\left(1 - s\frac{C_{gd1}}{g_{m1}}\right)}{1 + sa + s^2 b}$$



Chapter 4 Figure 14

$$a = R_s[C_{gs1} + C_{gd1}(1 + g_{m1}R_2)] + R_2(C_{gd1} + C_2)$$

$$b = R_s R_2 (C_{gd1}C_{gs1} + C_{gs1}C_2 + C_{gd1}C_2)$$

Assuming the poles are real and widely separated with $\omega_{p1} \ll \omega_{p2}$,

$$\omega_{p1} \cong \frac{1}{a} = \frac{1}{R_s[C_{gs1} + C_{gd1}(1 + g_{m1}R_2)] + R_2(C_{gd1} + C_2)}$$

$$\omega_{p2} \cong \frac{1}{\omega_{p1} b} \quad \omega_{p2} \cong \frac{g_{m1}C_{gd1}}{C_{gs1}C_{gd1} + C_{gs1}C_2 + C_{gd1}C_2}$$

$$\omega_z = -\frac{g_{m1}}{C_{gd1}}$$

Since $\omega_{p1} \ll \omega_{p2}, \omega_z$, a dominant-pole approximation may be applied for frequencies $\omega \ll \omega_{p2}, \omega_z$,

$$A(s) \cong \frac{A_0}{1 + \dfrac{s}{\omega_{p1}}} = \frac{-g_m R_2}{1 + s\{R_s[C_{gs1} + C_{gd1}(1 + g_{m1}R_2)] + R_2(C_{gd1} + C_2)\}}$$

$$\omega_{-3dB} \cong \omega_{p1} = \frac{1}{R_s[C_{gs1} + C_{gd1}(1 + g_{m1}R_2)] + R_2(C_{gd1} + C_2)}$$

If C_2 is large, we have

$$\omega_{p1} \cong \frac{1}{R_2 C_2}$$

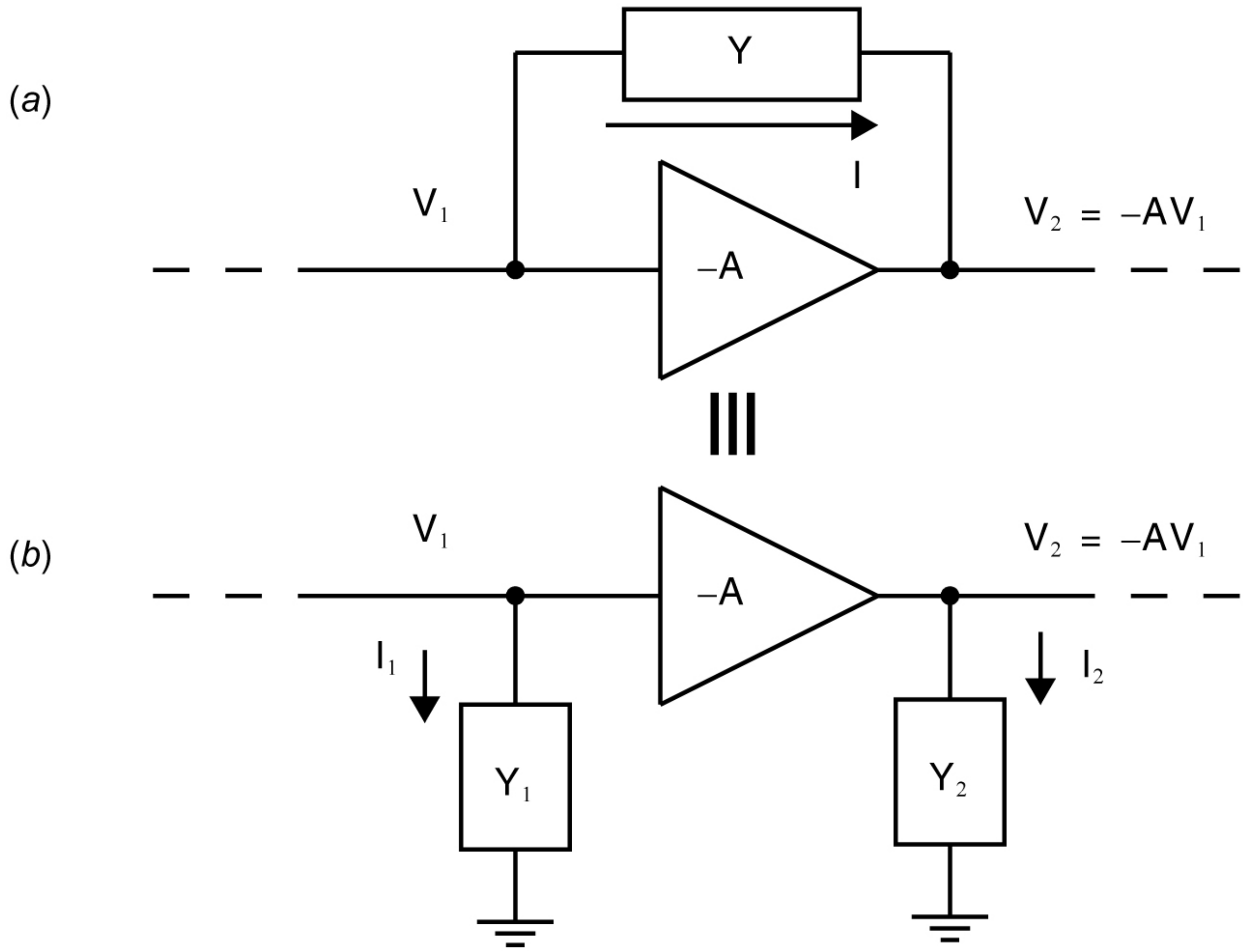
$$\omega_{p2} \cong \frac{C_{gd1}}{C_{gs1} + C_{gd1}} \frac{g_{m1}}{C_2}$$

Note that both ω_{p2} and ω_z are proportional to the transistor's transconductance g_{m1}; having a large transconductance is important in minimizing the detrimental effects of the second pole and the zero by pushing them to higher frequencies.

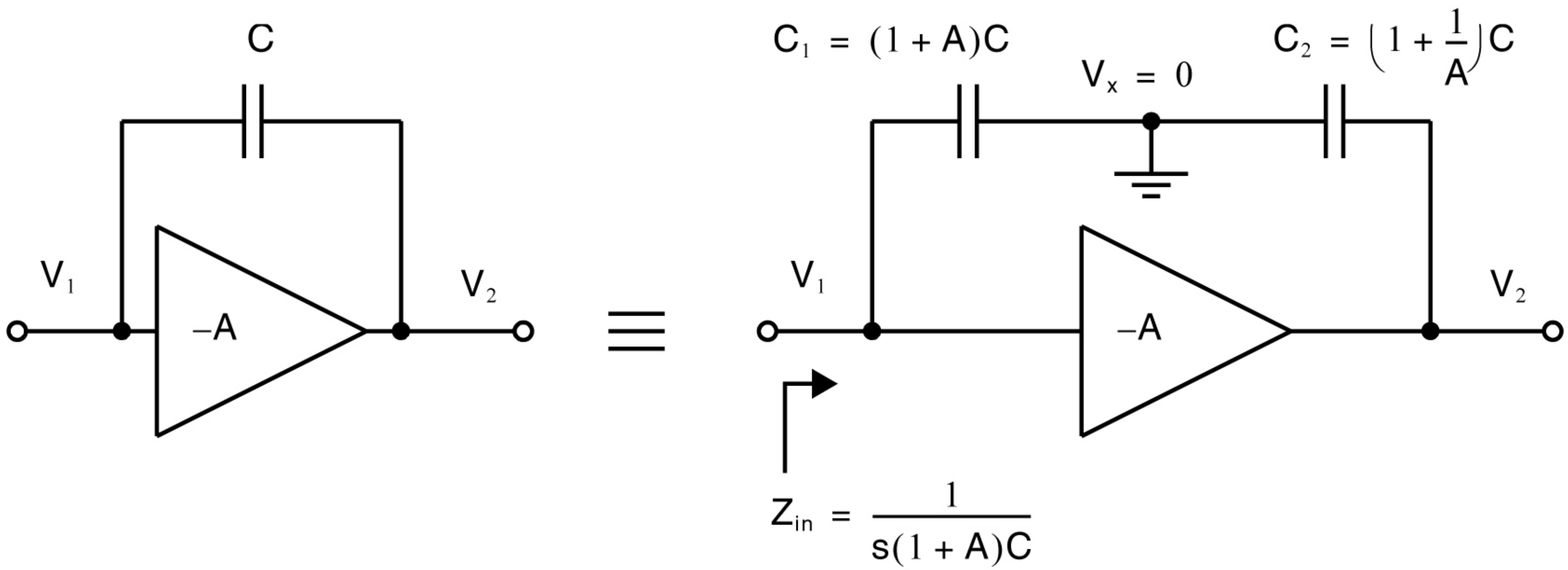
The high-frequency analysis of the common-source amplifier illustrates the critical importance of utilizing approximations in analog circuits, both to expedite analysis and to build intuition.

One more reason why analog design is tough.

4.2.3 Miller effect



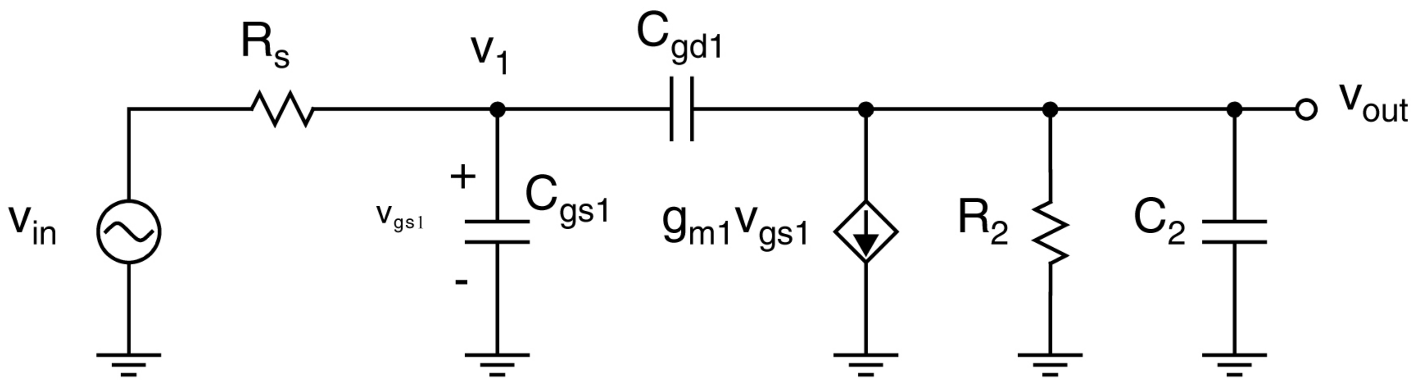
Chapter 4 Figure 15



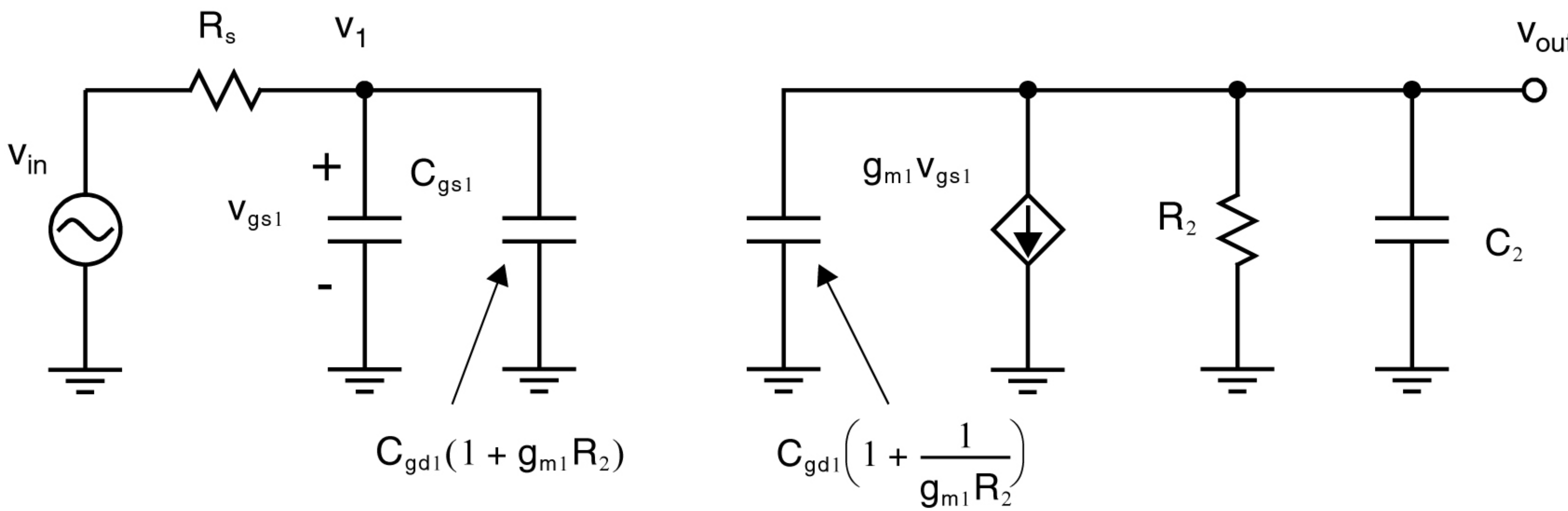
Chapter 4 Figure 17

Miller effect applied to CS amplifier

Miller effect allows one to quickly estimate the 3dB bandwidth in many cases.



Chapter 4 Figure 14



Chapter 4 Figure 18

4.2.4 Zero-value time constant method

Except Miller effect, the most common and powerful technique for frequency response analysis of complex circuits is the zero-value time constant analysis method.

It is very powerful in estimating a circuit's 3dB bandwidth with minimal complication and also in determine which nodes are most important.

Generally, the approach is to calculate a time-constant for each capacitor in the circuit by assuming all other capacitors are zero, then sum all time constants to estimate the 3dB bandwidth.

Detailed procedure:

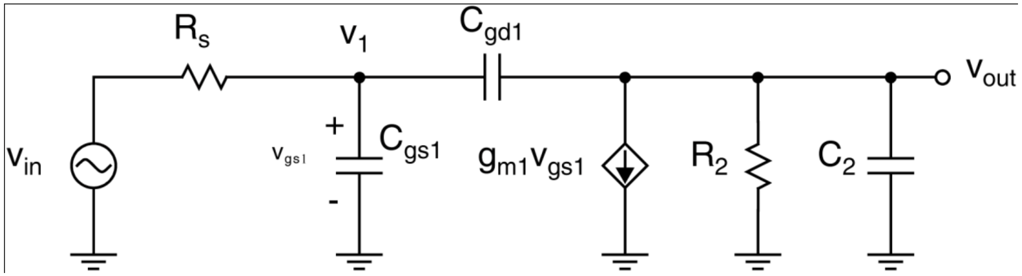
a. Set all independent sources to zero. That is, make all voltage sources into short circuits and all current sources into open circuits.

b. For each capacitor C_k in turn, with all other capacitors taken to be zero (making them open circuits), find a corresponding time-constant. To do this, replace the capacitor in question with a voltage source, and then calculate the resistance "seen by that capacitor," R_k, by taking the ratio of the voltage source to the current flowing from it. Note that in this analysis step, there are no capacitors in the circuit. The corresponding time-constant is then simply the capacitor multiplied by the resistance it sees: $\tau_k = R_k C_k$.

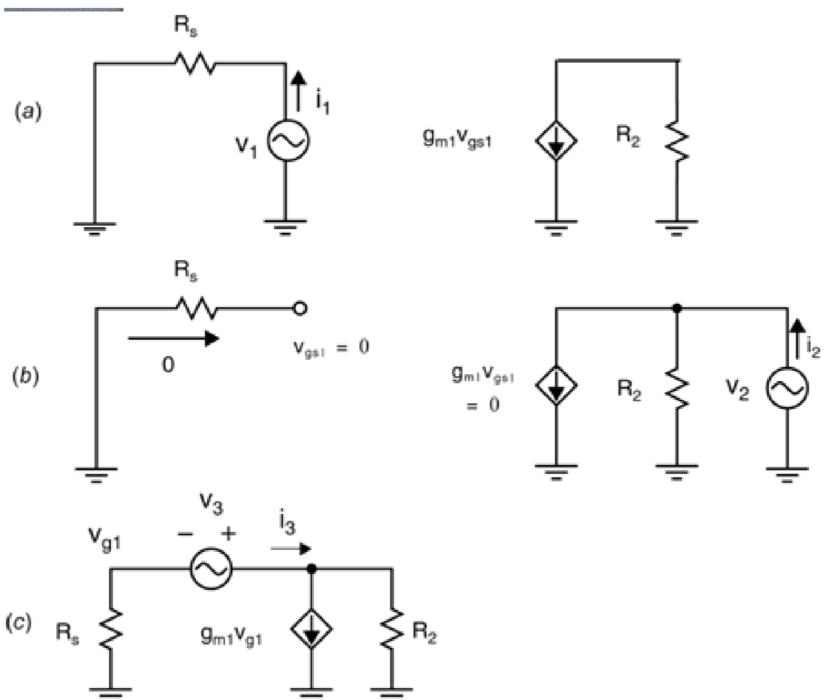
c. The –3dB frequency for the complete circuit is approximated by one over the sum of the individual capacitor time-constants.[6]

$$\omega_{-3dB} \cong \frac{1}{\sum \tau_k} = \frac{1}{\sum R_k C_k} \qquad (4.14$$

Example 4.9 (page 174)



Chapter 4 Figure 14



$$\tau_1 = R_s C_{gs1}$$

$$\tau_2 = R_2 C_2$$

$$\tau_3 = [R_s(1 + g_{m1}R_2) + R_2]C_{gd1}$$

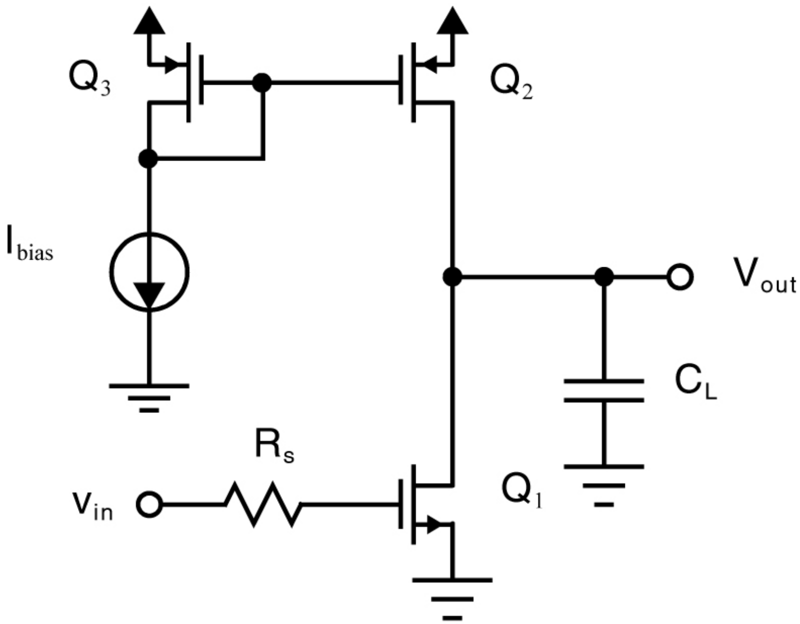
$$\omega_{-3dB} \cong \frac{1}{R_s C_{gs1} + R_2 C_2 + [R_s(1 + g_{m1}R_2) + R_2]C_{gd1}}$$

$$= \frac{1}{R_s[C_{gs1} + C_{gd1}(1 + g_{m1}R_2)] + R_2[C_2 + C_{gd1}]}$$

Chapter 4 Figure 19

The same as obtained previously

Design example 4.11 (page 177)



Chapter 4 Figure 13

$$\omega_{-3dB} \cong \frac{1}{R_s C_{gs1} + R_2 C_2 + [R_s(1 + g_{m1} R_2) + R_2] C_{gd1}}$$
$$= \frac{1}{R_s[C_{gs1} + C_{gd1}(1 + g_{m1} R_2)] + R_2[C_2 + C_{gd1}]}$$

We are to design the common source amplifier in Fig. 4.13 to provide a gain of 20 while driving a capacitive load of $C_L = 100$ fF with maximal bandwidth. The transistor parameters are those listed in Table 1.5 for the 0.18-μm CMOS technology. The input source resistance is $R_s = 40\ k\Omega$ and the supply voltage is 1.8 V. The ideal current source I_{bias} sinks 50 μA and the total power consumption is to be less than 1 mW.

In this example, the load capacitance is modest and source resistance is high, so C_{gd1} may become a major limitation of the bandwidth. This means that W_1 should be small.

So, given a current, V_{eff1} has to be relatively large: choose V_{eff1} to be 0.3V

Then suppose $L_1 << L_2$ so that $r_{ds2} >> r_{ds1}$ so $R_2 = r_{ds1}$ and $A_0 = -g_{m1} r_{ds1}$

$$A_0 \cong -g_{m1} r_{ds1} = -\frac{2 I_{D1}}{V_{eff,1}} \cdot \frac{1}{\lambda I_{D1}} = -\frac{2 L_1}{\lambda L_1 V_{eff,1}}$$

Design example 4.11 (page 177)

Then solve L_1 to be $L_1 = |A_0|\lambda L_1 V_{eff,1} = (20/2) \cdot 0.08\ \mu m/V \cdot 0.3\ V = 0.24\ \mu m$

Then note that increasing drain current of Q1 while keeping V_{eff1}=0.3V will increase g_{m1} and reduce r_{ds1} roughly in proportion, which results in about the same gain, but a smaller R_2 is achieved which increase 3db bandwidth. So, bandwidth is maximized by maximizing the drain current of Q1.

$$I_{D1} = \frac{1\ mW}{1.8\ V} - 50\ \mu A \cong 500\ \mu A$$

Then, we can compute the required gate width

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\frac{W_1}{L_1}V_{eff,1}^2$$

$$\Rightarrow W_1 = \frac{2 I_{D1} L_1}{\mu_n C_{ox} V_{eff,1}^2} = \frac{2 \cdot 500\ \mu A \cdot 0.24\ \mu m}{270\ \mu A/V^2 (0.3\ V)^2} \cong 10\ \mu m$$

To ensure $L_2 \gg L_1$, we can take $L_2 = 3L_1 = 0.72\mu m$

Then, we can arbitrarily and conveniently set $W_2 = 3W_1$

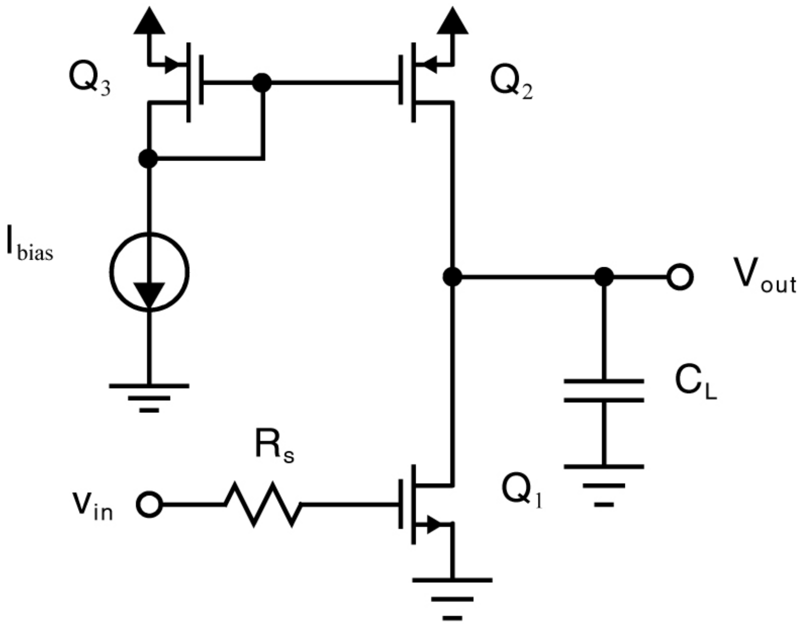
Finally, Q3 is sized to provide the desired current mirror ratio

$$L_3 = L_2 = 0.72\ \mu m$$

$$W_3 = W_2 \cdot \left(\frac{50\ \mu A}{500\ \mu A}\right) = 3\ \mu m$$

Then, need to make sure that all transistors are in active region

Design example 4.12 (page 178)



Chapter 4 Figure 13

$$\omega_{-3dB} \cong \frac{1}{R_s C_{gs1} + R_2 C_2 + [R_s(1 + g_{m1}R_2) + R_2]C_{gd1}}$$

$$= \frac{1}{R_s[C_{gs1} + C_{gd1}(1 + g_{m1}R_2)] + R_2[C_2 + C_{gd1}]}$$

We wish to design the common source amplifier in Fig. 4.13 with minimal power consumption while providing a 3-dB bandwidth of 5 MHz and a gain of at least 20 using the transistor parameters listed in Table 1.5 for the 0.18-μm CMOS technology with $C_L = 10$ pF and $R_s = 1$ kΩ. The ideal current source I_{bias} is 50 μA.

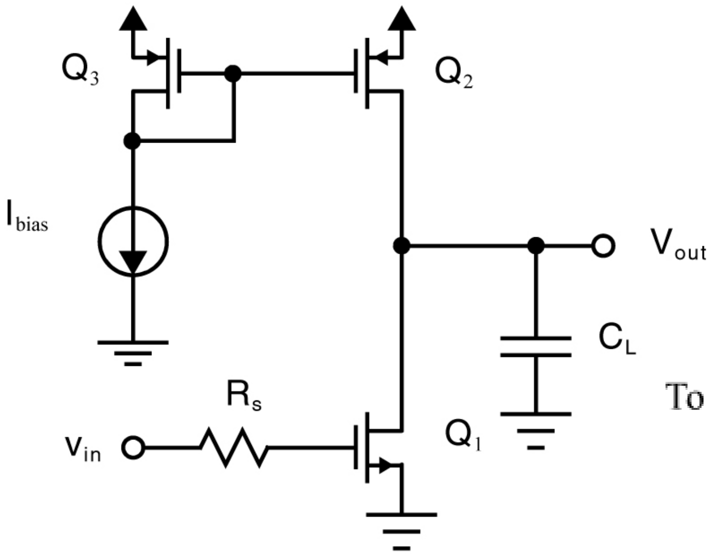
In this example, the load capacitance is very large and source resistance is small, so C_2 may become a major limitation of the bandwidth, so

$$\omega_{-3dB} \cong 1/(r_{ds,1} || r_{ds,2})C_L = 2\pi \cdot 5 \cdot 10^6 \text{ rad/sec}$$

$$(r_{ds,1} || r_{ds,2}) = 3.2 \text{ k}\Omega$$

Note that $(r_{ds,1} || r_{ds,2}) = 1/I_D(\lambda_1 + \lambda_2)$. To minimize I_D and, hence, power consumption we require large λ_1 and λ_2 which in turn demands small L_1 and L_2. Therefore, we use the minimum possible in this CMOS process.

Design example 4.12 (page 178)



Chapter 4 Figure 13

$$L_1 = L_2 = 0.18\ \mu m$$

$$\lambda_1 = \lambda_2 = (0.08\ \mu m/V)/0.18\ \mu m = 0.44\ V^{-1}$$

drain current is found. $I_D = \dfrac{1}{r_{ds}\cdot\lambda} \cong 350\ \mu A$ $\quad r_{ds}=2r_{ds1}$

To meet the gain requirement,

$$|A_0| \cong g_{m1}(r_{ds,1}||r_{ds,2}) = \frac{2I_D}{V_{eff,1}}\cdot\frac{1}{(\lambda_1+\lambda_2)I_D} = \frac{2}{V_{eff,1}}\cdot\frac{1}{(\lambda_1+\lambda_2)} > 20$$

$$\Rightarrow V_{eff,1} < \frac{2}{20(0.44\ V^{-1}+0.44\ V^{-1})} = 114\ mV$$

For some margin, we take $V_{eff,1} = 100\ mV$ and the resulting transistor width is

$$W_1 = \frac{2I_{D1}L_1}{\mu_n C_{ox} V_{eff,1}^2} = \frac{2\cdot 355\ \mu A\cdot 0.18\ \mu m}{270\ \mu A/V^2(0.1\ V)^2} \cong 47\mu m$$

The width of Q_2 may be chosen the same, $W_2 = 47\ \mu m$ and the size of Q_3 chosen to provide the correct current ratio in the current mirror formed by Q_2 and Q_3:

$$\frac{(W_3/L_3)}{(W_2/L_2)} = \frac{50\ \mu A}{350\ \mu A}$$

$$\Rightarrow L_3 = 0.18\ \mu m \ \text{ and } \ W_3 = 6.7\ \mu m$$

Comments

The above two design examples illustrate the manual analysis to provide an initial design solution, which thereafter needs to be refined iteratively using simulation. A number of challenges here:

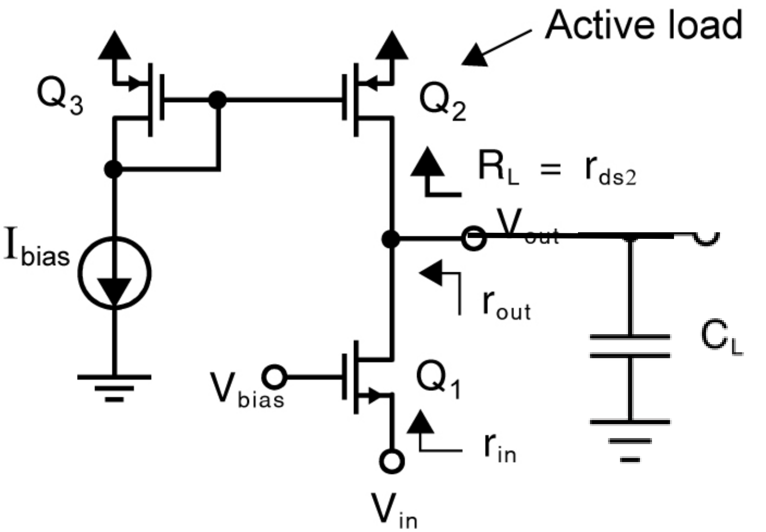
1. there is no guarantee that the initial solution is valid or good;
2. the refinement may take many many iterations until a good design is achieved;
3. at each iteration, what are not working or good in the circuit, what parameters to modify, and how to modify them requires in depth understanding of analog circuits.

What about those cases when it is hard to decide which capacitance dominates?

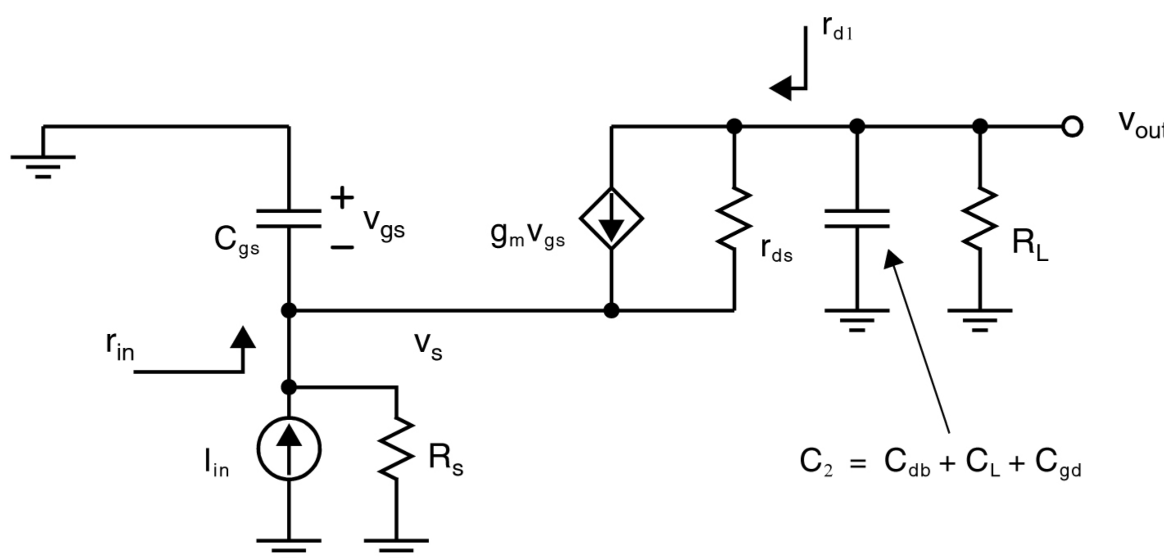
Experience counts here, after you had many designs and were aware of the biasing conditions, capacitance conditions?

The time domain response of common-source amplifier? (two widely spaced poles)

4.2.6 Common-gate amplifier



Chapter 3 Figure 09



Chapter 4 Figure 20

We estimate the time constant associated with C_{gs} (note that C_{gs} is connected between source and ground therefore may need to include C_{sb}).

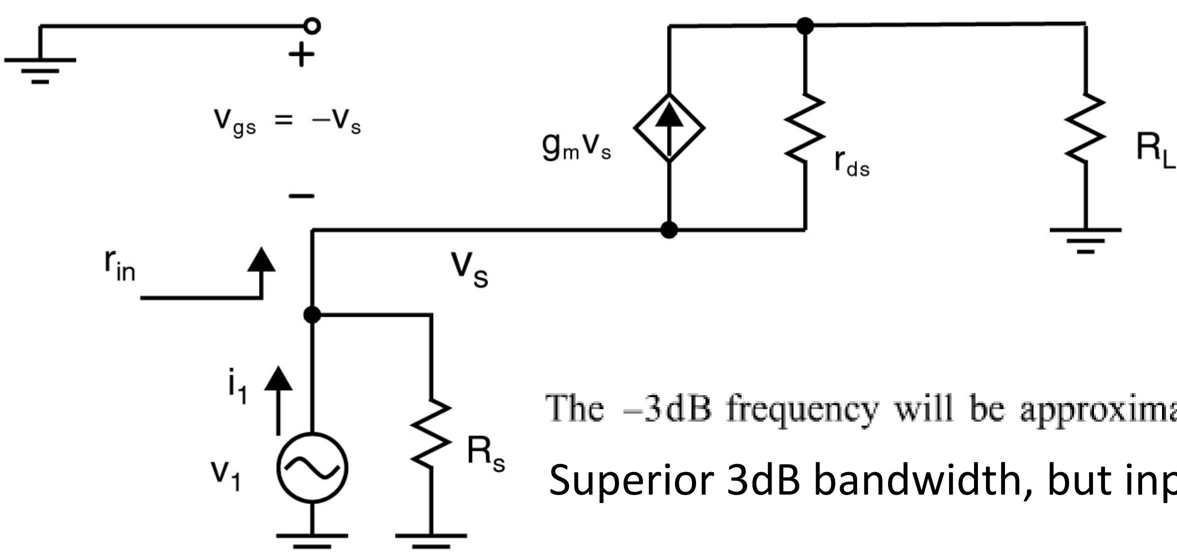
$$r_{in} \cong \frac{1}{g_m}\left(1 + \frac{R_L}{r_{ds}}\right)$$

$$R_1 = r_{in} \| R_s \cong \frac{1}{g_m} \| R_s \cong \frac{1}{g_m}$$ Under the assumption that R_L is not too much bigger than r_{ds}

$$\tau_1 = (r_{in} \| R_s)C_{gs} \cong \frac{C_{gs}}{g_m}$$ assumes $r_{in} \ll R_s$,

$$R_2 = R_L \| r_{d1} \qquad r_{d1} = r_{ds}(1 + g_m R_s)$$

$$\tau_2 = (R_L \| r_{d1})C_2 \cong R_L C_2$$



The -3dB frequency will be approximately given by $1/(\tau_1 + \tau_2)$.

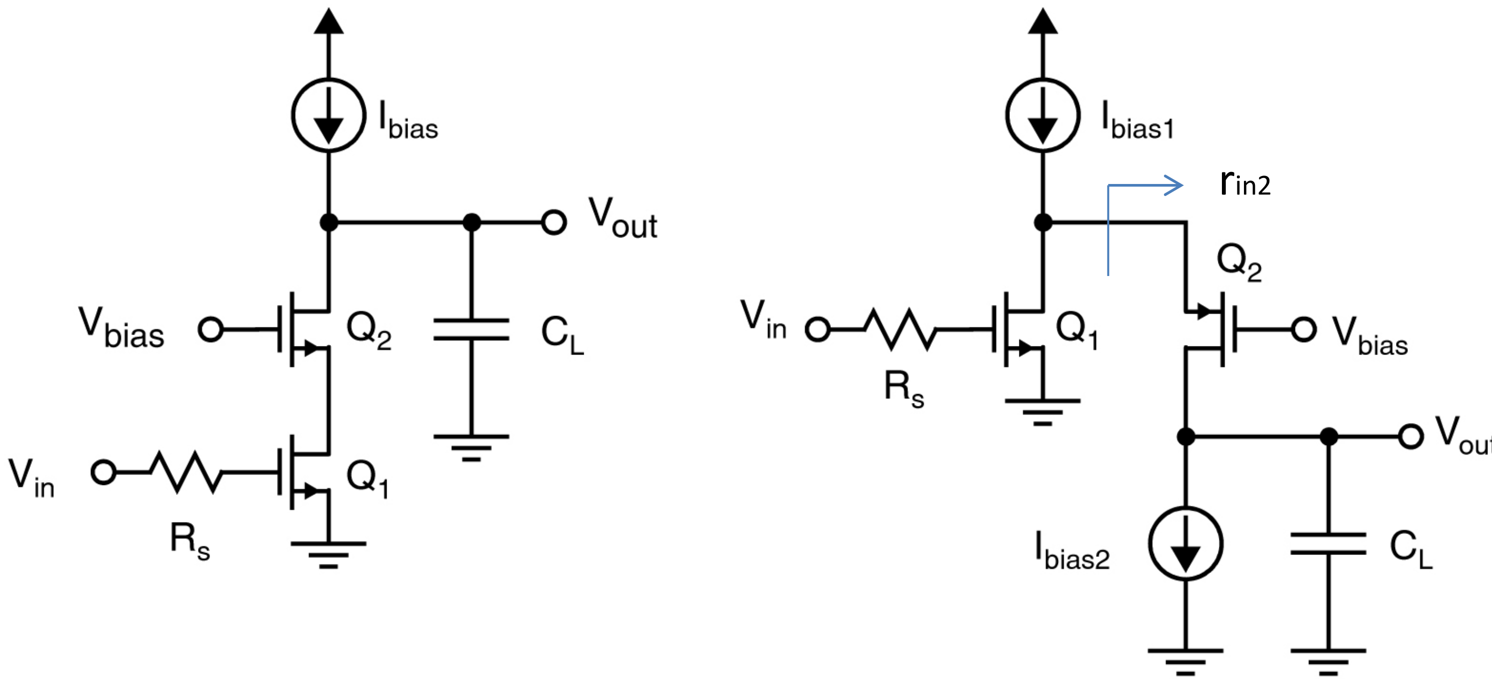
Superior 3dB bandwidth, but input impedance is too small.

Chapter 4 Figure 21

4.3 Cascode gain stage

Compared to CS amplifier, CG amplifier has much better 3dB bandwidth, but much smaller input impedance.

To achieve a good tradeoff, we can combine a CG amplifier with a CS amplifier.

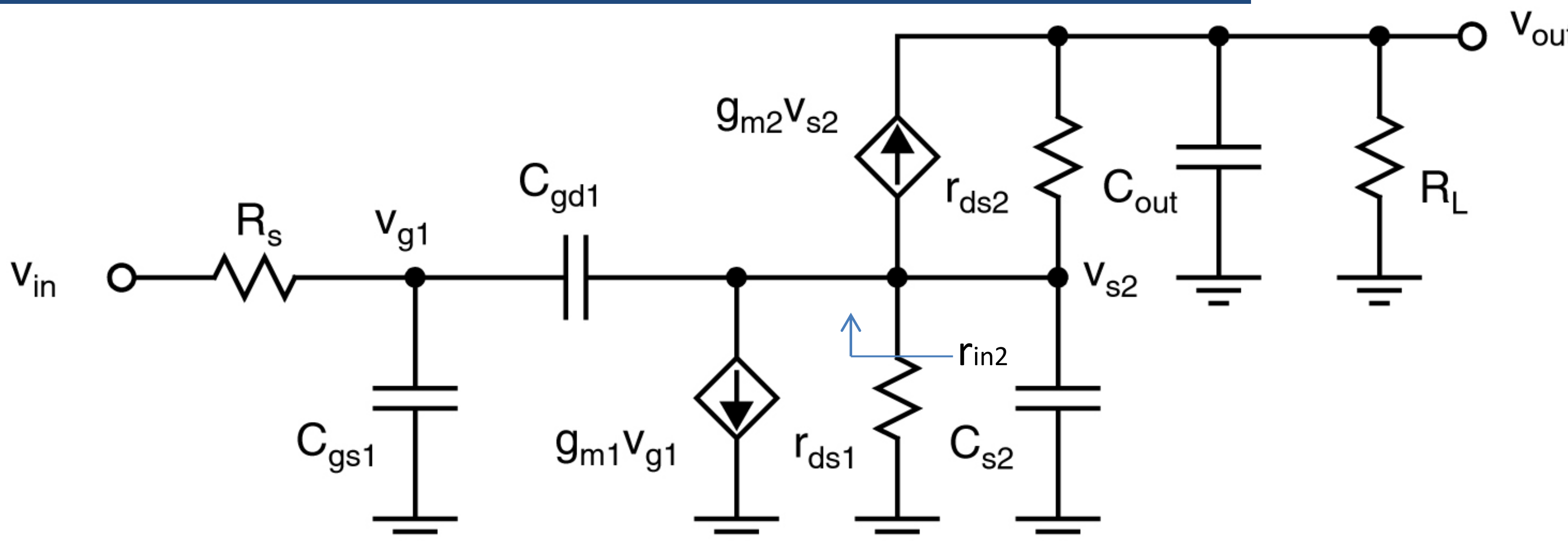


(a) (b)

Chapter 4 Figure 22

Telescopic Folded-cascode

Small-signal model for the cascode



Chapter 4 Figure 23

$C_{out}=C_{gd2}+C_{db2}+C_L+C_{bias}$

$C_{s2}=C_{db1}+C_{sb2}+C_{gs2}$

Using zero-value time constant method

$$\tau_{out} = R_{out}C_{out} = (r_{d2}||R_L)C_{out}$$

Please derive R_{out}

$$\tau_{gs1} = C_{gs1}R_s$$

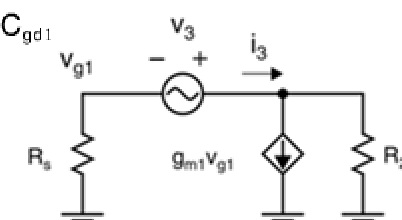
See slides 30 (Ch3)

The resistance seen by C_{s2} is the parallel combination of r_{in2} and r_{ds1}.

$$\tau_{s2} = (r_{in2}||r_{ds1})C_{s2} = \frac{C_{s2}}{g_{in2}+g_{ds1}}$$

The total resistance seen at the drain of Q1 is $r_{in2}||r_{ds1}$.

$$\tau_3 = [R_s(1+g_{m1}R_2)+R_2]C_{gd1}$$



$$\tau_{gd1} = \{R_s[1+g_{m1}(r_{in2}||r_{ds1})]+(r_{in2}||r_{ds1})\}C_{gd1}$$

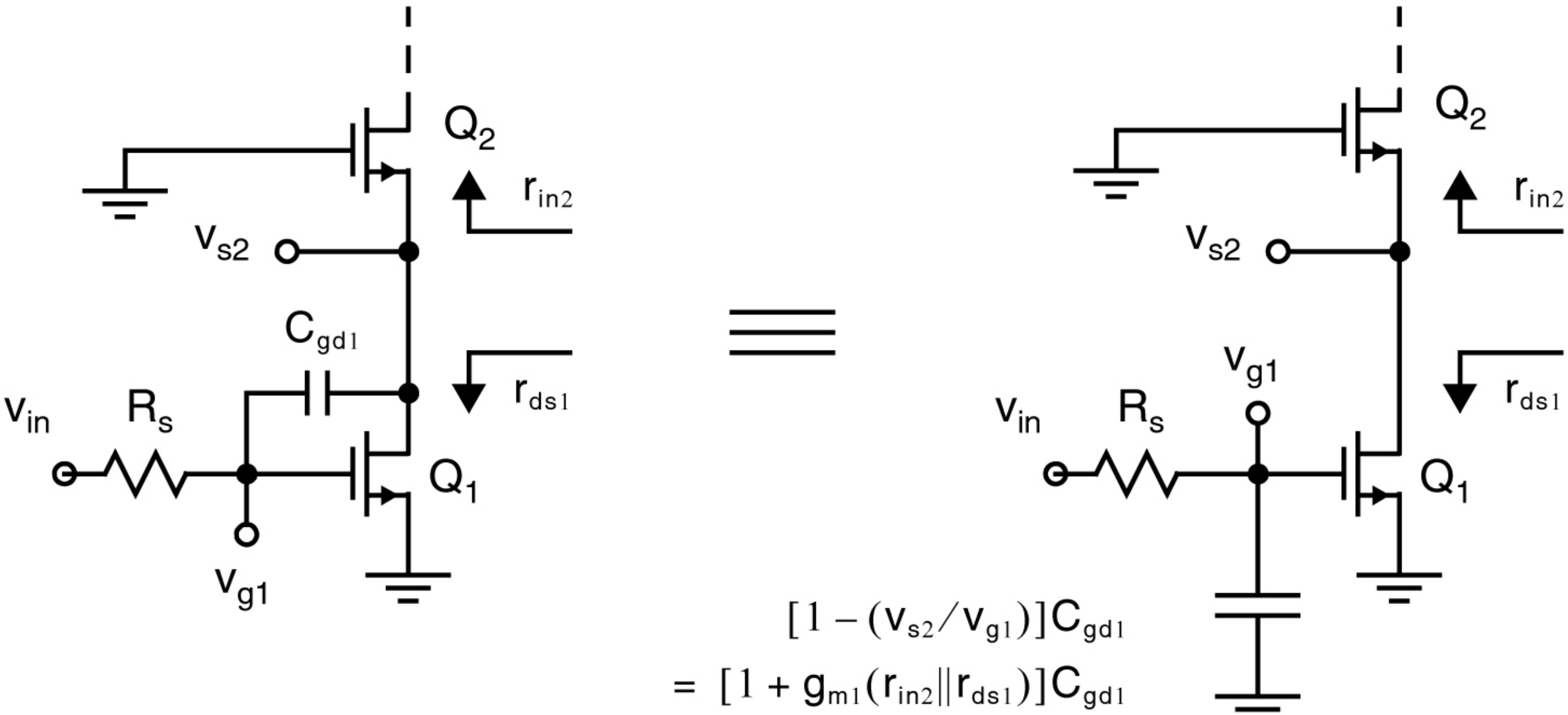
$$\cong R_s[1+g_{m1}(r_{in2}||r_{ds1})]C_{gd1}$$

See Slide 21

Miller effect

$$\tau_{total} = \tau_{out}+\tau_{gs1}+\tau_{s2}+\tau_{gd1}$$

On the Miller effect on C_{gd1}



$$[1 - (v_{s2}/v_{g1})]C_{gd1}$$
$$= [1 + g_{m1}(r_{in2} \| r_{ds1})]C_{gd1}$$

Chapter 4 Figure 24

Example 4.13, 4.14 (page 184-185)

Estimate the –3dB frequency of the cascode amplifier of Fig. 4.22(*a*). Assume that the current source I_{bias} has a high output impedance, on the order of $R_L \approx g_{m\text{-}p} r_{ds\text{-}p}^2$. Further assume that for all transistors, $g_m = 1$ mA/V, $r_{ds} = 100\ k\Omega$, $C_{gs} = 0.2$ pF, $C_{gd} = 15$ fF, $C_{sb} = 40$ fF, and $C_{db} = 20$ fF. The other component values are $R_s = 180\ k\Omega$, $C_L = 5$ pF, and $C_{bias} = 20$ fF.

$$C_{s2} = C_{db1} + C_{sb2} + C_{gs2} = 0.26\ \text{pF}$$

$$C_{out} = C_{gd2} + C_{db2} + C_L + C_{bias} = 5.055\ \text{pF}$$

when I_{bias} has high output resistance, $r_{in2} \approx r_{ds}$.

$$\tau_{gs1} = R_s C_{gs1} = 36\ \text{ns}$$

$$\tau_{gd1} \approx \frac{g_m r_{ds}^2}{2} C_{gd1} = 75\ \text{ns}$$ This approximation is valid since R_s is the same order as r_{ds}

$$\tau_{s2} \approx \frac{r_{ds}}{2} C_{s2} = 13\ \text{ns}$$

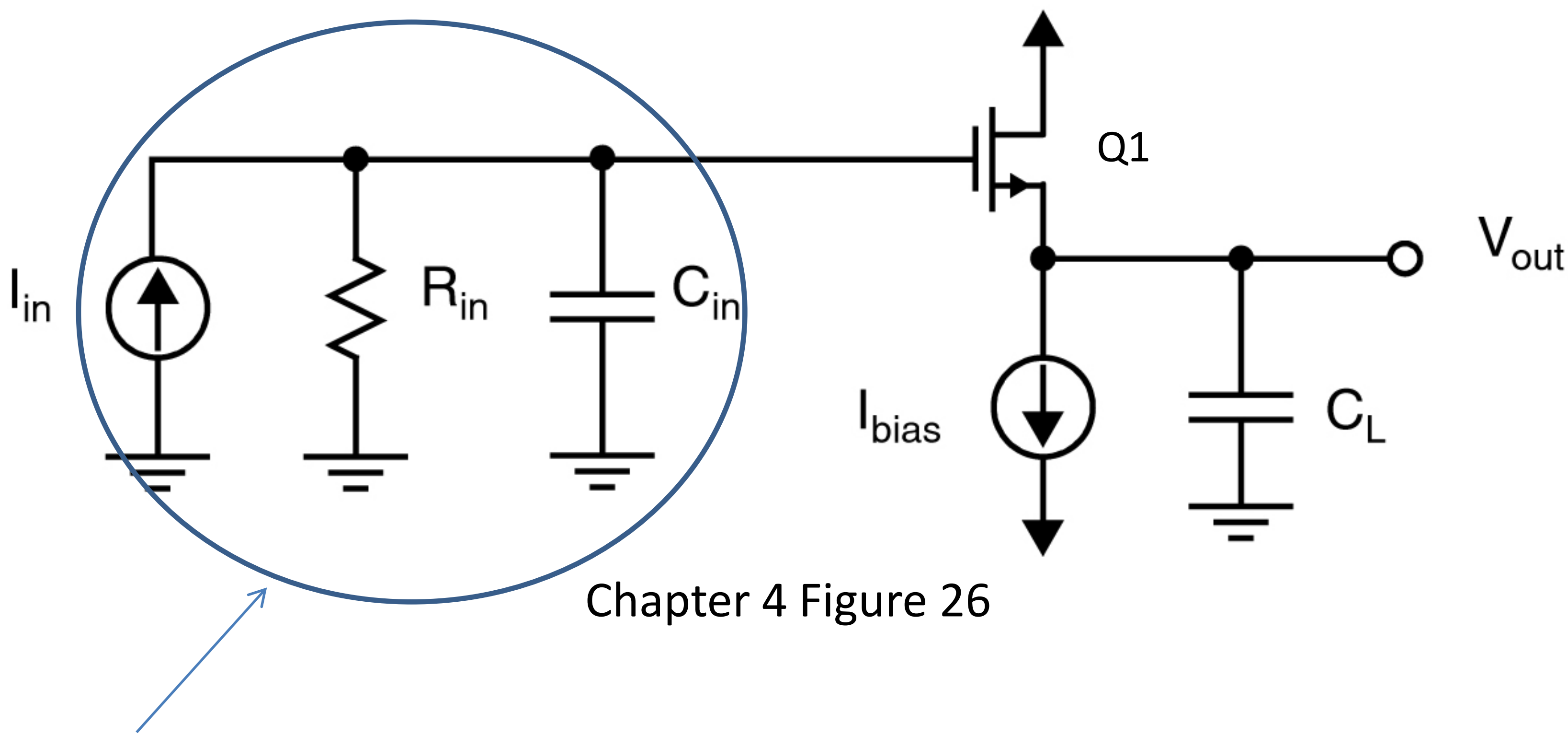
$$\tau_{out} \approx (g_m r_{ds}^2 || g_m r_{ds}^2) C_{out} = 25.3\ \mu\text{s}$$

$$\omega_{-3dB} \cong \frac{1}{\tau_{gs1} + \tau_{gd1} + \tau_{s2} + \tau_{out}} = 2\pi \times 6.3\ \text{kHz}$$

Recall that a large gain of the cascode amplifier requires the I_{bias} to have an output resistance on the order of $R_L \approx g_{m\text{-}p} r_{ds\text{-}p}^2$. In this case, and especially when there is also a large load capacitance C_L, the output time constant $\tau_{out} \approx g_m r_{ds}^2 C_{out}/2$ would dominate.

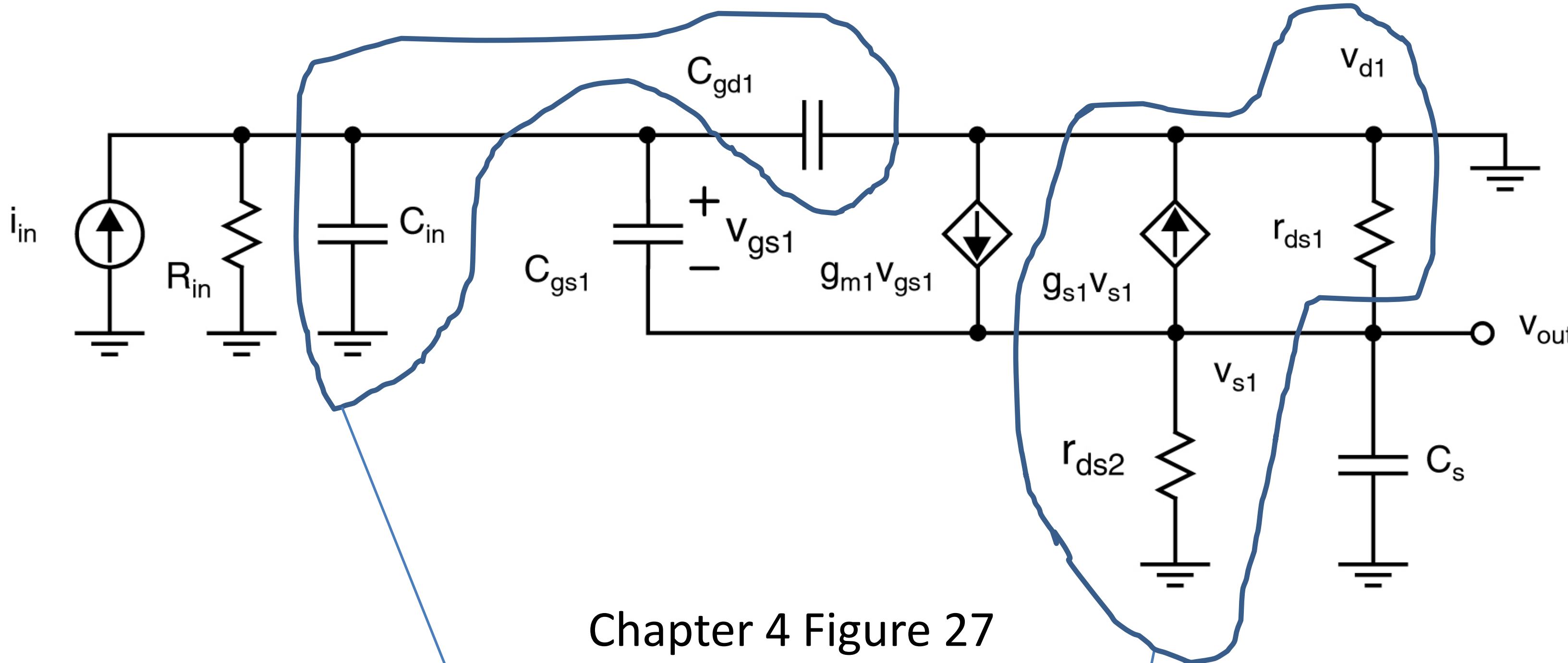
4.3 Source follower amplifier

The SF amplifier may have complex poles and therefore ringing and overshoot may happen for a pulse input.



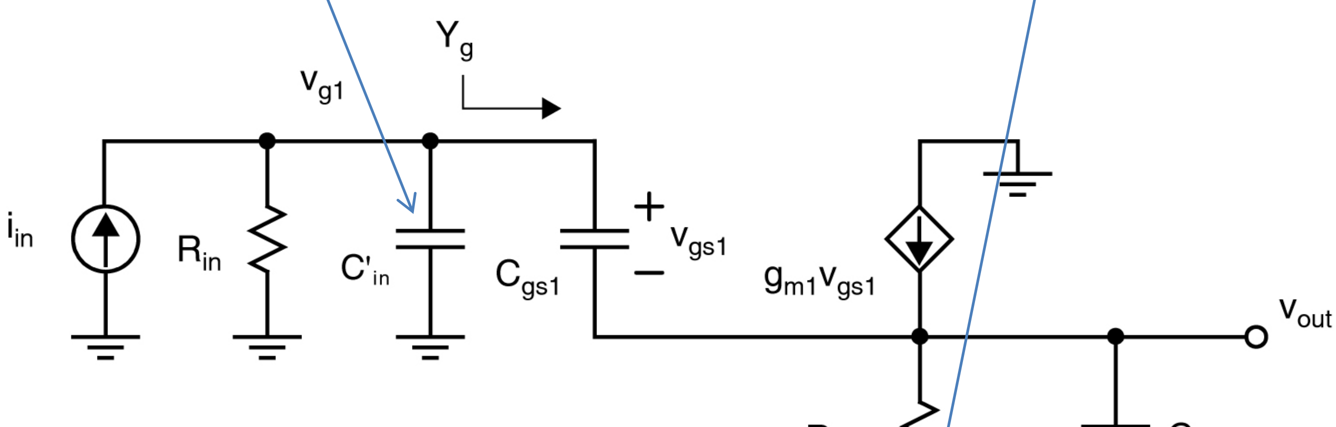
Chapter 4 Figure 26

Norton equivalent circuit



Chapter 4 Figure 27

$C_s = C_L + C_{sb1}$



$C'_{in} = C_{in} + C_{gd1}$

Chapter 4 Figure 28

$R_{s1} = r_{ds1} \| r_{ds2} \| (1/g_{s1})$

$$\frac{v_{out}}{v_{g1}} = \frac{sC_{gs1} + g_{m1}}{s(C_{gs1} + C_s) + g_{m1} + G_{s1}}$$

Next we find the admittance Y_g looking into the gate of Q1 (but not including C_{gd1} as it is already combined into C_{in}').

$$i_{g1} = (v_{g1} - v_{out})sC_{gs1} \qquad Y_g = \frac{i_{g1}}{v_{g1}} = \frac{sC_{gs1}(sC_s + G_{s1})}{s(C_{gs1} + C_s) + g_{m1} + G_{s1}}$$

$$i_{in} = v_{g1}(sC'_{in} + G_{in} + Y_g) \qquad \frac{v_{g1}}{i_{in}} = \frac{s(C_{gs1} + C_s) + g_{m1} + G_{s1}}{a + sb + s^2c}$$

$$a = G_{in}(g_{m1} + G_{s1})$$

$$b = G_{in}(C_{gs1} + C_s) + C'_{in}(g_{m1} + G_{s1}) + C_{gs1}G_{s1}$$

$$c = C_{gs1}C_s + C'_{in}(C_{gs1} + C_s)$$

$$A(s) = \frac{v_{out}}{i_{in}} = \frac{sC_{gs1} + g_{m1}}{a + sb + s^2c}$$

$$A(s) = A(0)\frac{N(s)}{1 + \frac{s}{\omega_0 Q} + \frac{s^2}{\omega_0^2}}$$

Recall Q<0.5 if no overshoot

$$\omega_0 = \sqrt{\frac{G_{in}(g_{m1} + G_{s1})}{C_{gs1}C_s + C'_{in}(C_{gs1} + C_s)}}$$

$$Q = \frac{\sqrt{G_{in}(g_{m1} + G_{s1})[C_{gs1}C_s + C'_{in}(C_{gs1} + C_s)]}}{G_{in}C_s + C'_{in}(g_{m1} + G_{s1}) + C_{gs1}G_{s1}}$$

If load capacitor Cs is very large compared to other capacitors such that

$$G_{s1} \ll g_{m1} \text{ and } C_s/g_{m1} \gg R_{in}(C_{gs1} + C_{in}') \qquad \text{that } Q \ll 0.5$$

then $$\omega_{p1} \cong Q\omega_0 = \frac{G_{in}(g_{m1} + G_{s1})}{G_{in}C_s + C'_{in}(g_{m1} + G_{s1}) + C_{gs1}G_{s1}} \cong \frac{g_{m1}}{C_s}$$

the zero-value time constant corresponding to the load capacitance C_s.

Note $\tau_{Cs} = C_s * R_s$, where $R_s \cong \frac{1}{g_{m1}} \,||\, r_{ds2} \cong \frac{1}{g_{m1}}$

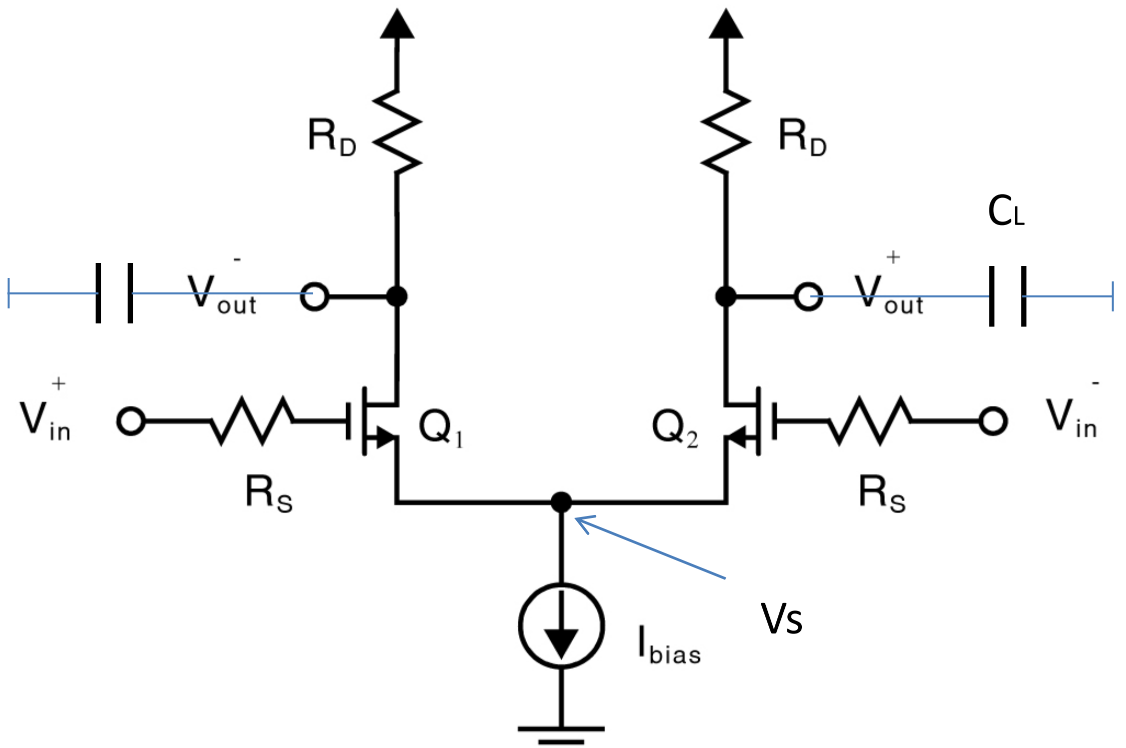
The resistance looking into the source of Q1

$$r_{in} = \frac{1}{g_{in}} = \left(\frac{1}{g_{m1}} \| \frac{1}{g_{s1}} \| r_{ds1}\right)\left(1 + \frac{R_L}{r_{ds1}}\right) \cong \frac{1}{g_{m1}}\left(1 + \frac{R_L}{r_{ds1}}\right) \cong \frac{1}{g_{m1}}$$ As $R_L = 0$ in this case

If Q is greater than 0.5, the poles will be complex-conjugate and the circuit will exhibit overshoot. For example, when C'_{in} and G_{s1} become small[8] then the circuit will have a large Q (i.e., large ringing) when G_{in} becomes small and $C_s \approx C_{gs1}$. Fortunately, the parasitic capacitances and output impedances in practical microcircuits typically result in only moderate overshoot for worst-case conditions.

4.5 Differential pair

When using T model for differential pair, the analysis may be simpler compared to the hybird-pi model.



Chapter 4 Figure 32

4.5.2 Symmetric differential pair

In the small-signal model, half circuit is analyzed to allow simpler analysis.

Also note that the V_s node is small-signal ground due to symmetry, so C_{sb1} and C_{sb2} can be neglected.

The half circuit corresponds to that of a CS amplifier, so the 3dB bandwidth is either

$$\omega_{-3dB} \cong \frac{1}{R_s[C_{gs1} + C_{gd1}(1 + g_{m1}R_2)]}$$

Where $R_2 = R_D || r_{ds1}$

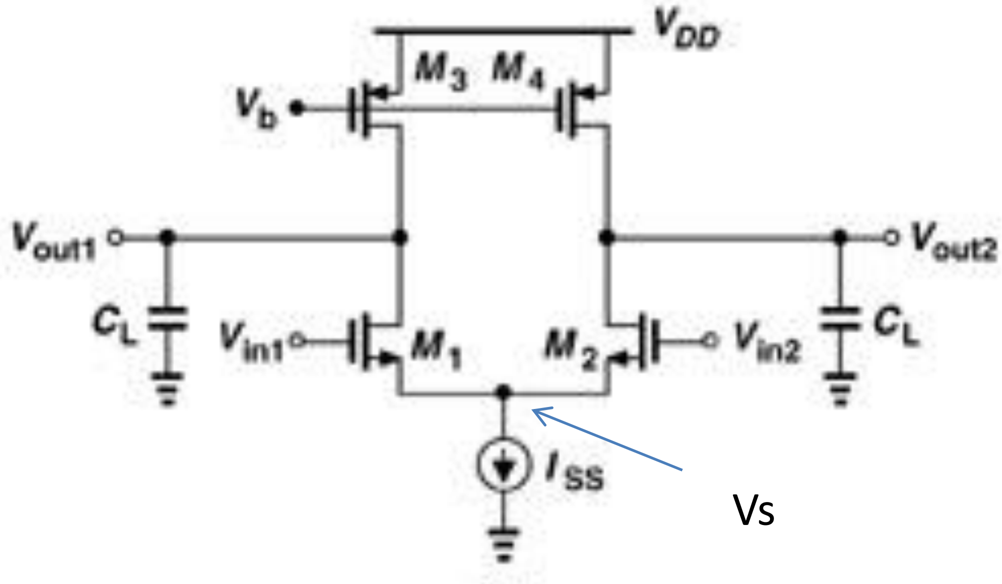
or $1/[R_2 * (C_{gd1} + C_{db1} + C_L)]$

Which one is the 3dB bandwidth depends on the C_L.

Active loaded differential pair

Again note that V_s is at small-signal ground, so half circuit can be used for analysis.

Again the load capacitor will determine which one is the 3dB bandwidth.



(b)

$$f_{b1} = \frac{1}{2\pi\,(r_{o1} // r_{o3})[C_L + C_{gd1} + C_{gd3} + C_{db1} + C_{db3}]}$$

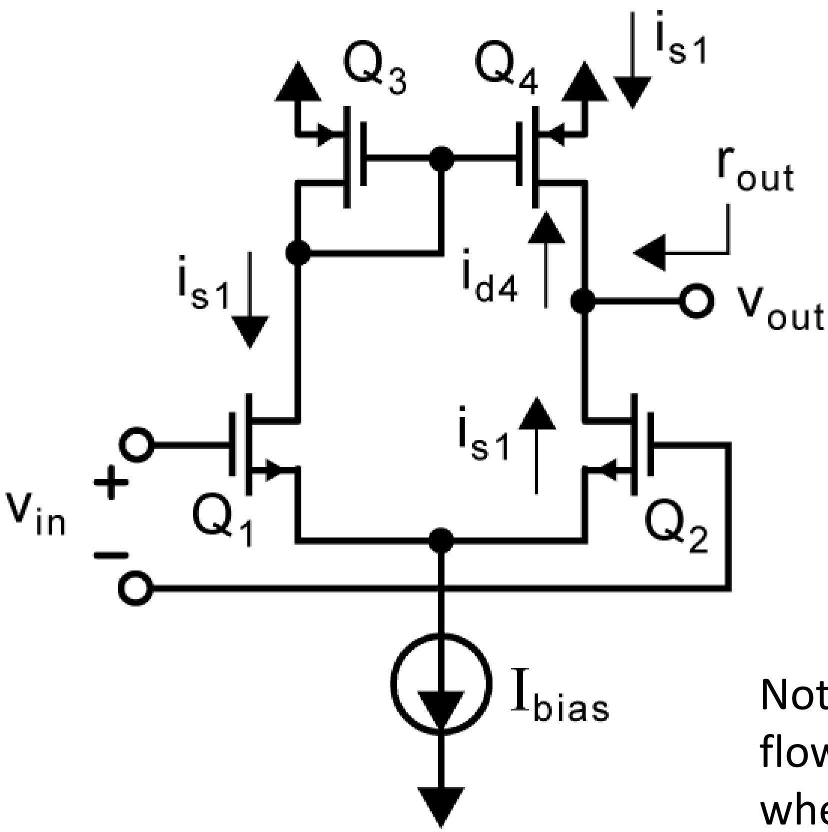
$$f_{b2} = \frac{1}{2\pi\, R_s[(1 + g_{m1}(r_{o1} // r_{o3}))C_{gd1} + C_{gs1}]}$$

Current-mirror loaded differential pair

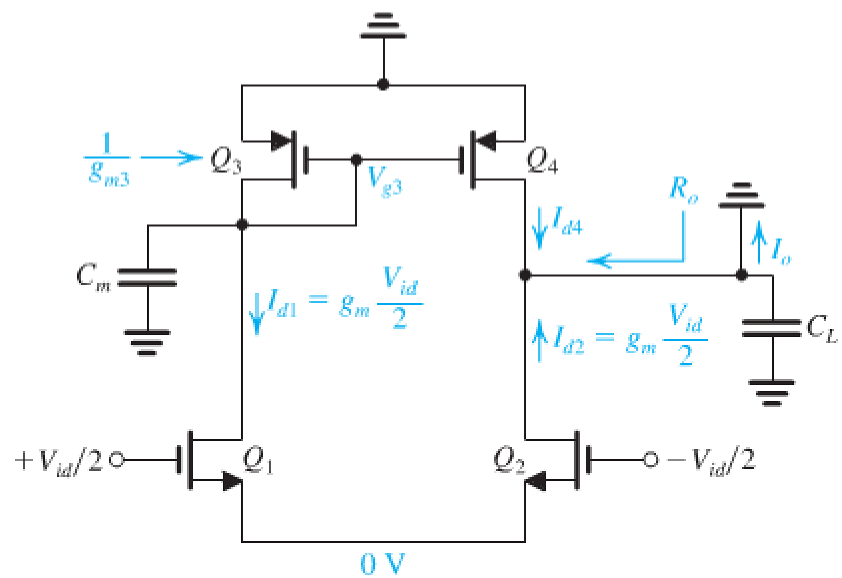
Capacitance at input node of the current mirror: $C_m = C_{gd1} + C_{db1} + C_{db3} + C_{gs3} + C_{gs4} + C_{gd4}$

Capacitance at the output node: $C_L = C_{gd2} + C_{db2} + C_{gd4} + C_{db4} + C_x$

From Miller effect



Chapter 3 Figure 19

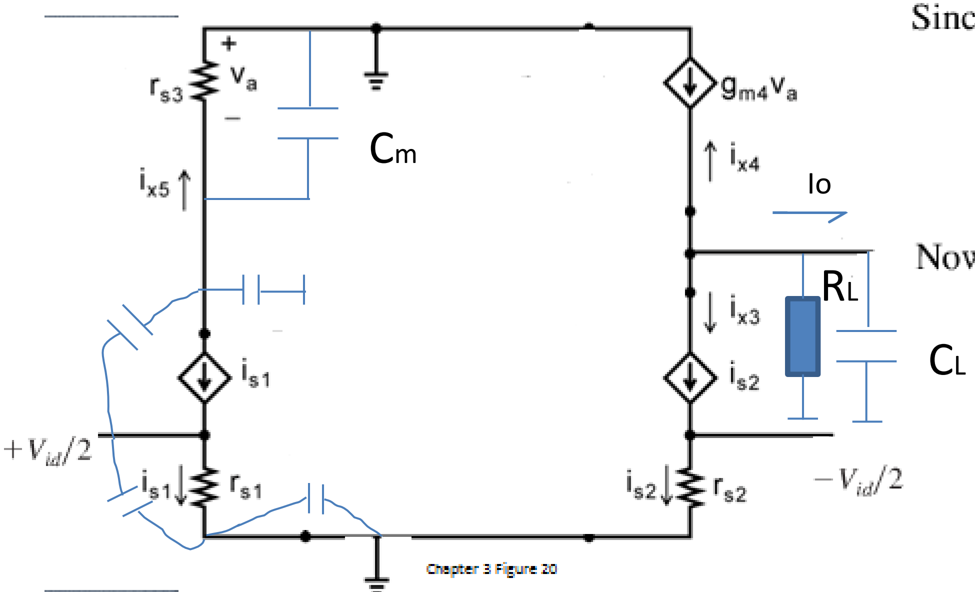


Note that Q1 will conduct an current of $g_m V_{id}/2$ flowing through Q3 (the parallel of $1/g_{m3}$ and C_m), where we neglected the effect of r_{ds1} and r_{ds2}, so

$$V_a = \frac{g_m V_{id}/2}{g_{m3} + sC_m}$$

Capacitance at input node of the current mirror: $C_m = C_{gd1} + C_{db1} + C_{db3} + C_{gs3} + C_{gs4}$

Capacitance at the output node: $C_L = C_{gd2} + C_{db2} + C_{gd4} + C_{db4} + C_x$



In response to Va transistor Q_4 conducts a drain current .

$$-I_{x4} = g_{m4}\ Va = \frac{g_{m4}\, g_m V_{id}/2}{g_{m3} + sC_m}$$

Since $g_{m3} = g_{m4}$, this equation reduces to

$$-I_{x4} = \frac{g_m V_{id}/2}{1 + s\dfrac{C_m}{g_{m3}}}$$

Now, at the output node the total output current

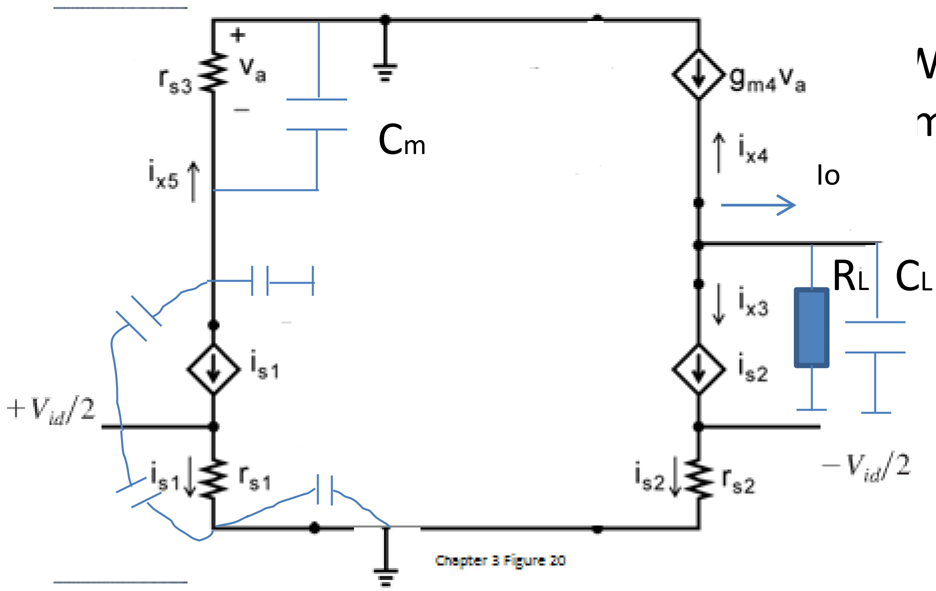
$$I_o = -I_{x4} - I_{x3}$$

$$= \frac{g_m V_{id}/2}{1 + s\dfrac{C_m}{g_{m3}}} + g_m(V_{id}/2)$$

$$G_m \equiv \frac{I_o}{V_{id}} = g_m \frac{1 + s\dfrac{C_m}{2g_{m3}}}{1 + s\dfrac{C_m}{g_{m3}}}$$

Capacitance at input node of the current mirror: $C_m = C_{gd1} + C_{db1} + C_{db3} + C_{gs3} + C_{gs4}$

Capacitance at the output node: $C_L = C_{gd2} + C_{db2} + C_{gd4} + C_{db4} + C_x$



Ve can them multiply G_m with the total load npedance to obtain the voltage Vo

$$V_o = I_o \frac{1}{\frac{1}{R_o} + sC_L}$$

$$= G_m V_{id} \frac{R_o}{1 + sC_L R_o}$$

$$\frac{V_o}{V_{id}} = (g_m R_o) \left[\frac{1 + s\frac{C_m}{2g_{m3}}}{1 + s\frac{C_m}{g_{m3}}} \right] \left(\frac{1}{1 + sC_L R_o} \right)$$

$$R_o = r_{o2} \parallel r_{o4}$$

Compared to the fully differential version, the current-mirror differential amplifier adds one more pole to the transfer function, therefore may significantly affect the frequency response.

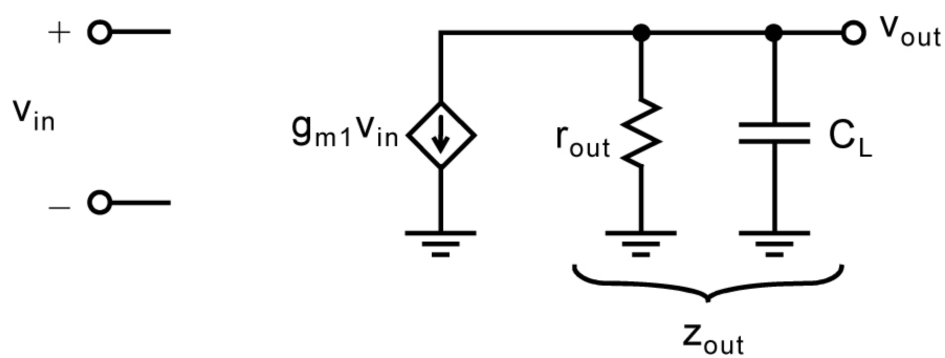
Simplified small-signal model for

If output load capacitance is dominated, then the following simple model can be used.

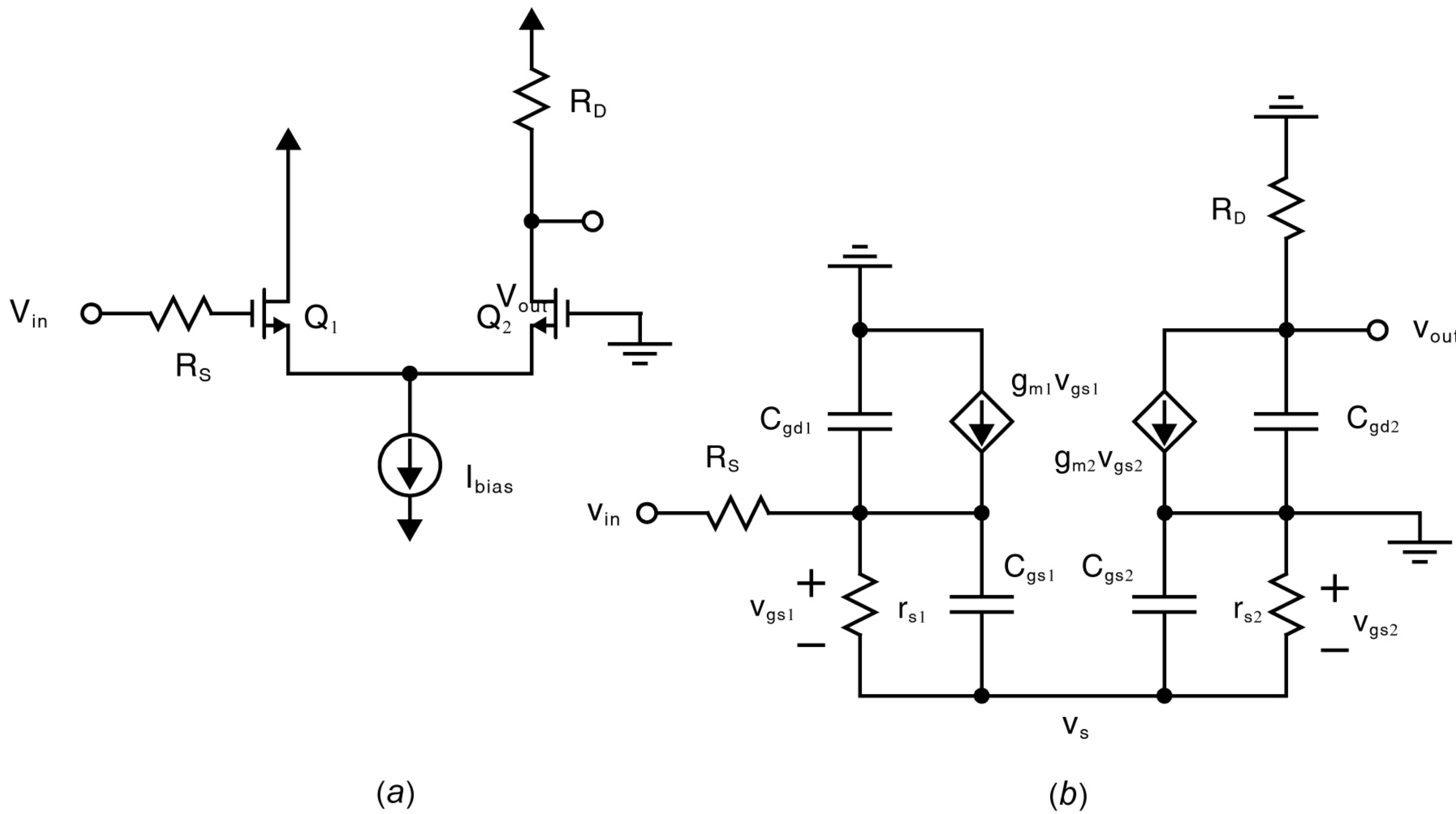
$$A_v = \frac{v_{out}}{v_{in}} = g_{m1} z_{out}$$

$$z_{out} = r_{out} \parallel 1/(sC_L).$$

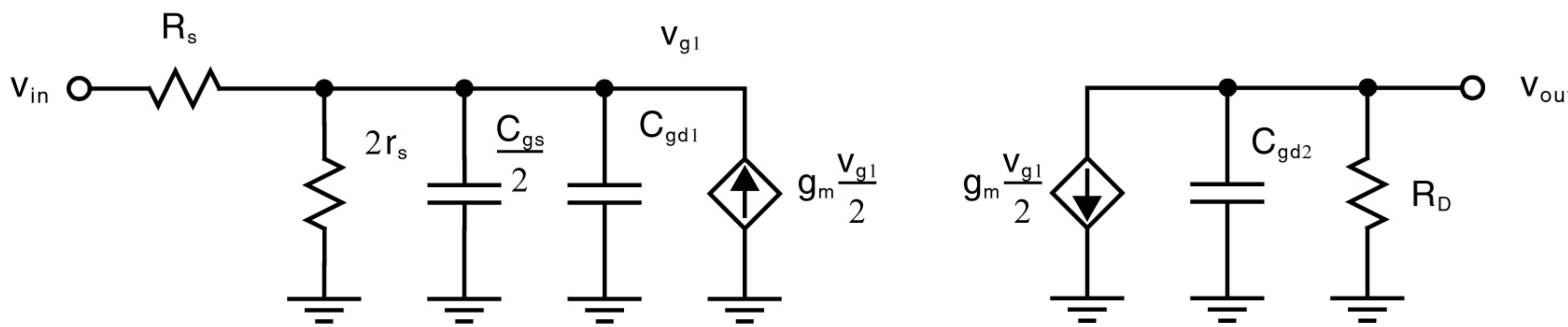
$$\omega_{-3dB} \cong \frac{1}{r_{out}C_L} = \frac{1}{(r_{ds2} \parallel r_{ds4})C_L}$$



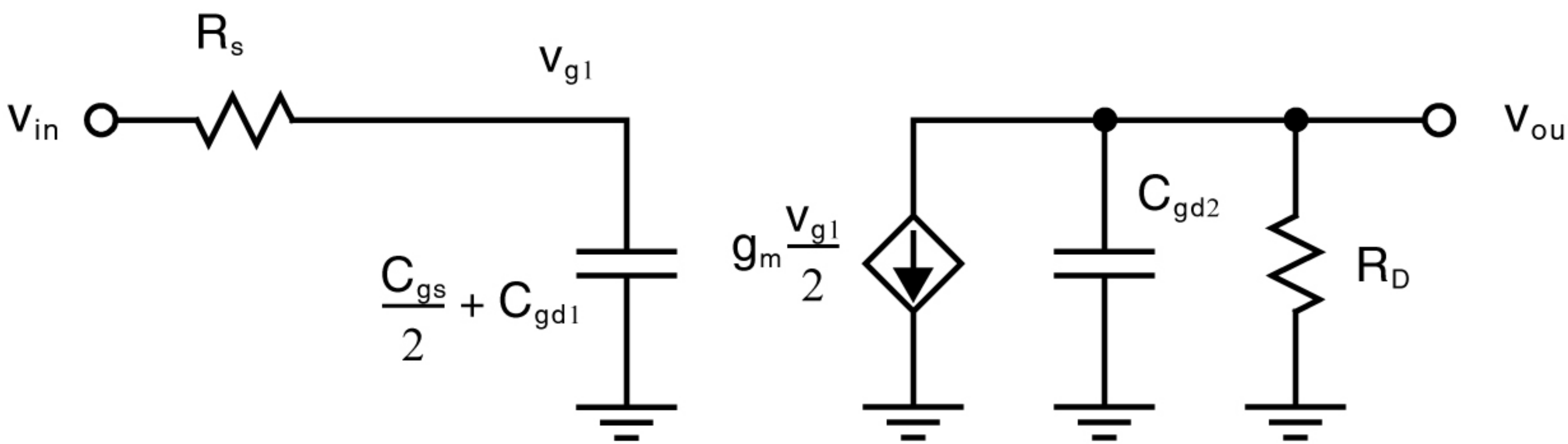
Chapter 4 Figure 37



Chapter 4 Figure 34



Chapter 4 Figure 35



Chapter 4 Figure 36