Recommended problems 1 — CS3512 — Fall '09

You should be *able* to do all the textbook exercises for Section 1.1. Look the problems over and do as many as you need. That does not mean you should spend your time doing them all (unless you need to)! You can start by checking yourself against the problems answered in the back of the book. (Marked by blue: you'll see.) If you have questions about other problems from the book, please send email or otherwise ask.

There are also questions raised in lecture notes (and left for you to answer). Worth considering.

Warning: If you have a first printing of the 2nd ed, the answer to problem 3(c) from Section 1.1 is wrong, in a mildly interesting way. The incorrect answer implies that 9 is not the product of two primes. Why might someone be tempted to say that? Because they think "two primes" means "two distinct primes", I'm guessing. (It doesn't in the context of the question, but that seems to be a subtle issue. As you know, a set has two elements iff it has two distinct elements. But more typically, if we have two things we do not assume they are distinct.)

Additional problems:

1 True or false? If $d \mid m$ and m is even, then d is even. Prove your answer.

2 True or false? If $d \mid m$ and m is odd, then d is odd. Prove your answer.

3 True or false? For integers d, a, b, if $d \mid ab$, then $d \mid a$ or $d \mid b$. Prove your answer.

Solutions to additional problems

Better to try them first yourself.

Also remember that there are many good solutions, but you could do worse than to consider why I might like mine...

1 True or false? If $d \mid m$ and m is even, then d is even. Prove your answer.

This is false. Take $d = 1$ and $m = 0$. Notice that $1 \mid 0$ and 0 is even but 1 is not even.

Remarks: This is a universal claim. That is, as usual when we make a claim involving new variables and do not use a quantifier (or otherwise explain the new variables), we should be understood to be making a universal claim. To show that such a claim is false, it is sufficient (indeed ideal) to produce a counterexample. (That is, a specific instance that falsifies the claim.) It can also help to explain your example a bit, depending on how easy and familiar the details are. (I think you can easily check that $1 \mid 0$, for instance.)

Further remarks: What if you don't notice that the claim is false? Well, if you're not sure, it is often fruitful to begin by looking for counterexamples. If you find none, you are likely to have gotten a better feeling for the claim, and can begin working on a proof for it. When a proof attempt fails, it often suggests where to look for a counterexample. And so on.

2 True or false? If $d \mid m$ and m is odd, then d is odd. Prove your answer.

True.

It is convenient to use the lemma discussed in the lecture notes:
for all integers n, n is even iff n is not odd.

Assume that $d \mid m$ and d is not odd. [NTS: m is not odd.] (Are you with me?)

Since d is not odd, we have by lemma that d is even.

So there are integers x, y s.t.

$$\begin{aligned} m &= dx \\ d &= 2y \end{aligned}$$

Therefore

$$\begin{aligned} m &= dx \\ &= 2yx \end{aligned}$$

And since yx is an integer, we can conclude that m is even, from which it follows by lemma that m is not odd.

Remarks: The difficulty here, it seems to me, is that the claim is hard to prove directly. (If you didn't before, perhaps you will try it now — and agree that it doesn't go well.) What I do is a variant of contrapositive (or indirect) proof. Here's one version of the story... In order to prove

$$\textbf{if } P \textbf{ and } Q, \textbf{ then } R \tag{1}$$

it suffices to prove

$$\textbf{if } P \textbf{ and not } R, \textbf{ then not } Q$$

because the two propositions are equivalent. Another version of the story... In an indirect proof of (1), we would assume **not** R in order to derive **not** (P **and** Q). And, in turn, in order to prove **not** (P **and** Q), it suffices to assume P in order to derive **not** Q. (Of course there are still other ways to explain this plan of the proof.)

3 True or false? For integers d, a, b, if $d \mid ab$, then $d \mid a$ or $d \mid b$. Prove your answer.

False. Notice that $4 \mid 6 \cdot 6$ but $4 \nmid 6$. (Do you follow?)